THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 51, PART 6 1 *November* 1939 NO. 288

THE GAMMA-RAY MEASUREMENT OF RADIUM

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Received 21 *November* 1938. *Read* 9 *June* 1939

ABSTRACT. In preference to calculation, experimental methods have been used for the determination of the obliquity and absorption corrections that must be applied to the measured content of a rectilinear radioactive source to find the true content. A two-stage bridge-type electrometer valve amplifier has been used to measure the ionization current produced in a cylindrical ionization chamber lined with brass. The effect on the corrections of variation of the following factors has been studied: (*a*) the distance between the source and the measuring instrument (from 10 to 60 cm.); (*b*) the active length of the container (from 0 to 10 cm.); (*c*) the equivalent lead wall-thickness of the cylindrical ionization chamber of volume 1 litre (from 6 to 12 mm.); (*d*) the wall-material (platinum, gold or silver), wall-thickness (from 0 to 2 mm.), and diameter of the radium-container; and (*e*) the thickness of the wooden source-holder. The effect of scattered and secondary radiation upon the measurements has been found to be quite small.

Percentage corrections derived from the experimental results are given in tabular form. These comprise a primary obliquity correction C_1, for the case of an unscreened source; a supplementary obliquity correction C_2, to allow for the oblique path of the measured radiation passing from the outer portions of the source through the container walls; two absorption corrections C_3, C_4, taking into account the absorption in the container wall and the source-holder respectively; and C_5, the usual correction for absorption in the radioactive salt itself.

The corrections have been found to be satisfactory by an experimental measurement of the strength of a radium-container previously calibrated by the National Physical Laboratory. Additional measurements have been made to demonstrate the variation of the mass absorption coefficient of the gamma-radiation with the atomic number of the material of the container screen and with the thickness of the lead filter. A description of a method of measurement of the internal diameters of cylindrical metal tubes is included.

§ 1. INTRODUCTION

GAMMA-RAY measurements of the radioactive content of various containers are effected by comparison with a standardized quantity of radium[1]. In practice, this method presents certain difficulties because several corrections, often quite large, must be applied to the measured or effective radioactive content to obtain the true content. A knowledge of the effective content only is of limited value, since this quantity depends definitely upon the experimental condi-

tions of measurement; for example, a change in the wall-thickness of the ionization chamber may alter one of the corrections necessary to convert the measured to the true content by as much as 4 per cent. The magnitude and uncertainty of the corrections increase when the containers to be compared differ widely in construction and dimensions. With the increasing attention paid to the precise calculation of dosage in gamma-ray therapy, accurate knowledge of the true content of radium-containers and radon-containers becomes more and more desirable.

The various national and other laboratories apply to their measurements[2, 28] corrections which are determined for their own particular measuring instruments. This Laboratory is responsible[29] for radium standardization measurements, particularly with regard to the issue of radon for treatment purposes from five centres in Australia. Substandard tubes in sets ranging from 1 to 20 mg., calibrated against the Laboratory national radium substandard, are used for measuring the content of radon-filled gold capillary tubing. The specifications of these three types of containers which have to be intercompared are given in table 1, and indicate the widely divergent dimensions and screenages for which corrections must be known.

Table 1

Specifications	Containers			
	National substandard	20-mg. lab. substandard	1-mg. lab. substandard	Radon-filled capillary
Total length (mm.)	29·2	28·0	21·5	Up to 100
Active length (mm.)	28·7	20·0	10·0	Up to 100
External diameter (mm.)	3·6	3·45	1·65	0·82
Internal diameter (mm.)	3·1	1·2	0·6	0·15
Wall-thickness (mm.)	0·27	1·0	0·5	0·33
Wall-material	Glass	Gold-platinum	Platinum	Gold
Mode of filling	Partially-filled	Tightly-packed	Tightly packed	Radon
Content	20·3 mg.	20 mg.	1 mg.	Up to 60 mc.

As complete details of the measuring instruments and corrections in use elsewhere were not available, it was considered desirable to determine these for the experimental equipment of the Laboratory. A full description of the methods employed and results obtained will be given, in order to enable radium-containers to be calibrated with an ionization chamber of easily reproducible construction by applying tabulated correction factors without the necessity for further work. Further, the opportunity was taken to compare the accuracy and convenience of the gamma-ray electroscope with the valve-amplifier method of measurement.

The problem outlined may, in theory, be solved by calculation, but in practice serious mathematical difficulties are encountered, and the time involved would probably exceed that required for an experimental solution. Further, unless all factors are taken into proper account, the calculated corrections may be in error.

The two most important factors to be considered are: (*a*) the spatial distribution of the radioactive source relative to the measuring instrument, and (*b*) absorption of the measured radiation in the wall of the radium-container.

§§ 2 to 6 of this paper are devoted to the effect of (*a*), while §§ 7 to 16 are concerned with (*b*).

The magnitude of the correction for the spatial distribution of the source, i.e. the obliquity correction, is determined by the following geometrical and experimental conditions: (*a*) the shape and size of the radioactive source, (*b*) the distance between the source and the ionization chamber; (*c*) the orientation of the source relative to the ionization chamber; (*d*) the shape and size of the chamber; (*e*) the material and thickness of the wall of the chamber; (*f*) the materials and thicknesses both of the source-support and of the source itself.

It was not, of course, necessary or desirable to vary all of the above conditions which were therefore limited in the following respects. The radioactive sources, which were all rectilinear and of small internal diameter, were placed in a V-shaped wooden holder in the broadside-on position relative to a cylindrical ionization chamber, having a volume of about 1 litre, lined with brass, and covered with lead. The remaining experimental conditions were varied.

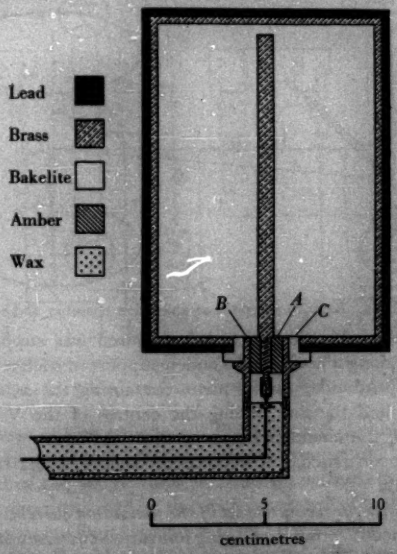

Figure 1. Cross section of cylindrical ionization chamber.

§ 2. MEASURING APPARATUS

2·1. *Ionization chamber.* The ionization chamber employed was constructed from cylindrical brass tubing with circular pieces of sheet brass screwed to the ends; see figure 1. The central electrode was a brass rod supported from the bottom of the chamber by means of an amber button *A*, a brass guardring *B*, and a bakelite

insulating bush *C*. Experiments showed that saturation conditions were obtained for the strongest sources with a potential-difference of 160 v. across the chamber; so in practice the wall was maintained at about 240 v. above earth-potential, and the guardring *B* was earthed so that any leakage across the surface of the amber button *A* was minimized. Any leakage which occurred between the outer portion of the chamber and the guardring would not, of course, be measured as part of the ionization current. The lead sheathing covered all sides of the ionization chamber, the specification of which is as follows: shape, cylindrical; metal, brass enclosed in lead sheath; internal diameter, 9·50 cm.; internal length, 14·09 cm.; internal volume, 994 c.c.; diameter of central electrode, 0·635 cm.; length of central electrode, 13·60 cm.; thickness of brass walls, 3·30 mm.; thickness of lead walls, 3·32 mm.; equivalent total lead thickness of walls calculated on a density basis, 5·78 mm.

By adding two annular lead sheaths in turn, the wall-thickness of the chamber was varied so as to have the following lead equivalents: chamber no. 1, 5·8 mm.; chamber no. 2, 9·0 mm.; chamber no. 3, 12·3 mm.

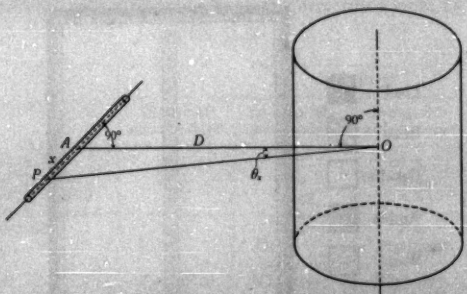

Figure 2. Relative positions of ionization chamber and source.

2·2. *Source-holder*. The source to be measured was supported in a wooden V mounting sliding along a horizontal wooden support in such a way that the axis of the source was perpendicular to the plane containing the axis of the cylindrical ionization chamber and the line joining the centres of the V mounting and the chamber, the axis of the chamber being vertical; see figure 2. The whole source-support system was constructed of light cedar wood in order to reduce scattering effects to a minimum.

2·3. *Valve amplifier for measurement of the ionization current*. 2·31. *Circuit*. The valve-amplifier method [3–21]* of measuring ionization current was employed because of its speed and the diminished exposure to the operator. A detailed description of the amplifier is considered to be unnecessary, since the latter is similar to others which have been described.

The circuit, figure 3, follows the valve-bridge principle with two stages of amplification, the output current being read with a microammeter. The usual

* References given in a very complete list by Horton (3) have been omitted from the bibliography of this paper.

devices for compensating for fluctuations in the various battery voltages were incorporated in the circuit. The components of the circuit were as follows: C, ionization chamber; R, $2{\cdot}2 \times 10^{11}$ Ω. liquid resistance; P, potentiometer leads; A, Weston microammeter and shunt; V_1, V_2, Philips electrometer valves, type 4060; V_3, V_4, Mullard valves, type P.M. 2A; rheostats 1 and 7, 0–10,000 Ω.; rheostat 9, 0–0·5 Ω.; rheostat 11, 0–7 Ω.; resistance 2, 10,000 Ω.; resistance 3, 25,000 Ω.; resistance 4, 20,000 Ω. tapped every 2000 Ω.; resistance 5, 50,000 Ω.; resistance 6, 65,000 Ω.; resistance 12, 40,000 Ω.; resistance 13, 15,000 Ω.; resistance 15, 5000 Ω.; resistance 16, 4000 Ω.; slide wire 8, 1 Ω.; slide wire 10, 1·5 Ω.; plug-in resistance box 14, 0–27,000 Ω.; box 17, 0–2700 Ω.

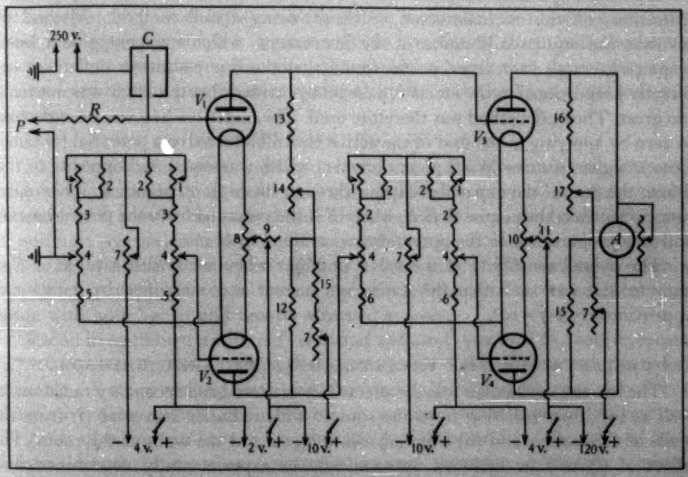

Figure 3. Wiring diagram of electrometer valve amplifier.

2·32. *Construction of high resistances.* The amplifier could be used as an electrometer in either of two ways: (1) the ionization current could be used to build up a charge on the grid of the active valve V_1, and measurements made of the rate of drift of the output current as read on the microammeter; or (2) the ionization current could be passed through a high resistance R, and the potential drop across R applied to the grid of the valve V_1. Method (2) was adopted because speed and utility were preferred to extremely high sensitivity.

For a convenient sensitivity, R should be at least 10^{10} Ω., and should remain reasonably free from temperature changes and secular changes. For this reason the liquid mixtures of Gyemant[22] appeared preferable to xylol-alcohol mixtures[23] or other types of resistance[24, 25, 26]. The liquid was placed in a container of soda glass tubing with platinum wire leads sealed in the ends, the whole external surface being

carefully cleaned, dried and coated with a film of non-hygroscopic insulating wax (cerise AA) to eliminate electrical leakage.

2·33. *Details of construction and operation.* In the construction of the amplifier, the methods and precautions recommended by other writers were generally followed, but cerise AA wax was used to insulate the lead from the ionization chamber to the grid of the active electrometer valve. After the instrument had been balanced experimentally by the cut-and-try method described by Nottingham[3], it was found to respond reasonably quickly to changes of ionization current and to give steady readings. Small adjustments of the setting of the several variable resistances were found to be necessary only at infrequent intervals.

With the amplifier it was possible to read ionization currents either by (*a*) reading deflections on the microammeter, or by (*b*) using a null method. Method (*a*) involves the regular calibration of the instrument, which may not give a linear response, though in practice it was found that the microammeter deflection was directly proportional to the ionization current provided that the latter was not made too great. The null method was therefore used. The meter reading was brought back to zero by applying to the grid of the active electrometer valve a potential (obtained from a high-resistance Wolff potentiometer) which was equal and opposite to that due to the flow of the ionization current through the high resistance R. The ionization current was then equal to E/R, where E is the potential from the potentiometer, and was proportional to the potentiometer reading in ohms.

The overall sensitivity of the valve amplifier when used with a value of R of $2{\cdot}2 \times 10^{11}\ \Omega$. was such that the ionization current was magnified by a factor of approximately $1{\cdot}7 \times 10^{8}$.

§ 3. TESTS FOR THE PRESENCE OF SCATTERED RADIATION

The ionization chamber may be affected by scattered and secondary radiation, as well as by direct radiation from the source. This radiation can arise (i) from the walls of the source, and (ii) from adjacent objects and the walls of the room. The effect of (i) will be included automatically in experimentally determined wall absorption or obliquity corrections. If the effect of (ii) is appreciable, however, corrections determined by one observer with a given disposition of apparatus relative to external scattering objects may be inapplicable to measurements made by another observer working under altered conditions.

The apparatus used in this work was situated in the centre of a room 40 ft. long, 30 ft. wide, and 20 ft. high, and consisted essentially of the amplifier cabinet (34 in. × 21 in. × 14 in.) and a wooden block (11 in. × 8 in. × 4 in.) supporting two lead blocks (9 in. × 8 in. × 2 in. and 9 in. × 5 in. × 1½ in.) situated behind the ionization chamber. The wooden source-holder arm (30 in. × 2 in. × 1 in.) and the ionization chamber were at a height of about 2 ft. above the top of a wooden table, which was itself about 3 ft. above floor-level.

A series of tests was carried out with large blocks of material placed in various positions adjacent to the source and ionization chamber in order to determine whether radiation from solid objects, such as the wooden table top, would affect the ionization measurements to any appreciable extent.

The result of these tests indicated that the radiation scattered from such objects in the vicinity of the ionization chamber and the source did not increase the ionization current by more than 3 parts in 10,000.

A further experiment was made to determine the amount of scattered radiation entering the chamber from a comparatively massive piece of metal (gold) placed in the line of the axis of, and directly touching, the end of the source. The mean of 20 potentiometer readings was found to be 4584·1 Ω. with the container only and 4586·4 Ω. with the container plus the scatterer. Since the mean deviation of the readings was approximately ±3·7, i.e. 0·08 per cent, this difference was not significant.

The tests described above might be expected to yield negative results, however, for two reasons: (i) very little gamma radiation is scattered in directions other than those making angles of a few degrees with the direction of the primary beam; and (ii) part of the secondary radiation is of longer wave-length than the primary radiation and would, therefore, be more readily absorbed in the lead wall of the ionization chamber.

§ 4. EXPERIMENTAL RESULTS

4·1. *Method of obtaining readings.* A series of measurements of the ionization current was made with an elementary source placed in turn at the centre, and at distances of 2, 4 and 6 cm. on each side of the centre of the source-holder. The order of positions was varied to eliminate errors due to any drift in the zero of the instrument. The distance of the source-holder from the centre of the ionization chamber was varied in steps from 10 to 60 cm. For one chamber (no. 1) measurements were made with both lightly and heavily screened sources.

Radon in equilibrium with radium B and radium C was used as a source because much greater strengths (in millicuries per cm.) were obtained than with the radium available. A set of readings occupied about 2 hr., during which time the effect of radioactive decay, which amounted to 1·5 per cent, was eliminated by reversing the order of taking readings throughout the run. The details of the sources are as follows: length of container, 10 mm.; external diameter, 0·815 mm.; internal diameter, 0·142 mm.; wall-thickness, 0·336 mm.; metal, 24-carat gold. Details of the screen are as follows: length of sheath, 17·6 mm.; external diameter, 4·763 mm.; internal diameter, 1·179 mm.; wall-thickness, 1·792 mm.; metal, 24-carat gold. The total wall-thickness of the composite source was 2·128 mm.

The effect of stray ionization in the valve amplifier was eliminated by taking readings with the ionization chamber disconnected from the apparatus, and thus the ionization produced in the chamber alone could be determined. In general, 20 independent readings were taken for each position of the source, the latter being removed and replaced later in each position. The mean departure from the mean value for any position was about 0·4 per cent for source-to-chamber distances greater than 30 cm., and about half that at smaller distances, where larger ionization currents were produced.

4·2. *Results.* The mean experimental results obtained from independent series of measurements made with an elementary radon source screened with 0·336 mm. of gold with the three ionization chambers, and the composite source screened with

2·128 mm. of gold with ionization chamber no. 1, are given in table 2. The mean of the values obtained at equal distances from the centre of the source-holder has been aken, so that each result is the mean of about 40 observations. The quantity tabulated is the measured value of the ratio I_x'/I_0', where I_0' is the measured ionization with the centre of the 1-cm. elementary source in the centre of the source-holder, and I_x' is the measured ionization with the centre of the same source at a distance x cm. from the centre of the source-holder.

Table 2. Values of $(I_x'/I_0')_A$

D (cm.)	x (cm.)	Inverse-square value	Mean values for chamber no. 1		Mean values for chamber no. 2	Mean values for chamber no. 3
			Wall-thickness 0·3 mm. gold	Wall-thickness 2·1 mm. gold		
10	2	0·9617	0·9674	0·9628	0·9662	0·9671
	4	0·8624	0·8816	0·8666	0·8792	0·8826
	6	0·7353	0·7647	0·7401	0·7654	0·7693
15	2	0·9826	0·9830	0·9820	0·9839	0·9850
	4	0·9337	0·9362	0·9327	0·9387	0·9391
	6	0·8622	0·8686	0·8573	0·8709	0·8730
20	2	0·9901	0·9892	0·9903	0·9902	0·9899
	4	0·9616	0·9617	0·9608	0·9624	0·9632
	6	0·9175	0·9178	0·9132	0·9208	0·9198
30	2	0·9956	0·9954	0·9963	0·9938	0·9950
	4	0·9826	0·9795	0·9825	0·9805	0·9822
	6	0·9616	0·9582	0·9599	0·9598	0·9624
40	2	0·9975	0·9964	0·9952	—	0·9953
	4	0·9901	0·9853	0·9877	—	0·9844
	6	0·9780	0·9740	0·9765	—	0·9710
50	2	0·9984	—	—	—	0·9962
	4	0·9936	—	—	—	0·9926
	6	0·9858	—	—	—	0·9850
60	2	0·9989	0·9970	0·9965	0·9965	0·9988
	4	0·9956	0·9948	0·9926	0·9929	0·9911
	6	0·9901	0·9892	0·9865	0·9887	0·9856

For comparison purposes, corresponding values of I_x'/I_0' are included in the table, calculated from the inverse-square law by means of the following formula, which is easily deduced for the case of a rectilinear source having an active length of 1 cm.:

$$\frac{I_x'}{I_0'} = \frac{1}{1+R} + 0{\cdot}41667\,\frac{R}{D^2} - 1{\cdot}1667\,\frac{R^2}{D^2} - 0{\cdot}1403\,\frac{R}{D^4}, \qquad \ldots\ldots(1)$$

where D cm. is the distance between the centres of the source-holder and the chamber, x cm. is the lateral displacement of the source, and $R=(x/D)^2$.

§ 5. DERIVATION OF CORRECTION FACTORS

5·1. *Correction for absorption in container screen and source-holder.* An examination of the results given in table 1 for ionization chamber no. 1 shows significant differences between readings obtained with sources having different wall-thicknesses, owing to the increased absorption in the container screen and the source-holder when the measured beam of radiation travels obliquely through, as is the case for

larger values of the lateral displacement x. The magnitude of this effect may be calculated with sufficient accuracy by the following approximate method.

In figure 2 let O be the centre of the chamber, A the centre of the radium holder and source, and P the point on the source considered. Then if D cm. is the distance OA, x cm. the distance AP, s_1 cm. the thickness of the container screen, and θ_x the angle AOP, it will be seen that radiation from P passes through an average thickness of metal $s_1 \sec\theta_x$ or $s_1\sqrt{(1+x^2/D^2)}$.

Let $(I_x'/I_0')_T$ be the true value of the ratio of the effects of a short unscreened source lying at P, and lying at the centre A, while $(I_x'/I_0')_A$ is the apparent value of a similar ratio for a similar screened source. Then we have approximately

$$(I_x'/I_0')_A = (I_x'/I_0')_T\, e^{-(\mu_1 s_1 + \mu_2 s_2)(\sec\theta_x - 1)}, \qquad \text{.....(2)}$$

where μ_1 is the apparent absorption coefficient of the radiation in the wall material of the container, μ_2 is the apparent absorption coefficient of the radiation in the material of the source-holder, and s_2 cm. is the thickness of the source-holder traversed by the central ray from a source in the position $x=0$.

For the measurements which have been described, values of θ_x, s_1 and s_2 were known, and μ_1 and μ_2 were obtained experimentally (see §§7–16 of this paper), so that values of $(I_x'/I_0')_T$ were calculable.

By means of equation (2), corrections were applied to the values of $(I_x'/I_0')_A$ from table 2 to give values of $(I_x'/I_0')_T$ which are set out in table 3. The largest correction was 3·35 per cent for the case of the screened source with D equal to 10 cm. and x equal to 6 cm. After correction, the agreement between the results for the two independent series with containers of different wall-thicknesses, measured with the same chamber no. 1, is close. This agreement is considered to provide sufficient justification of the adequacy of the above approximate method of correction, since the range of wall-thickness covered is large (from 0·3 to 2·1 mm. of gold).

The mean values of $(I_x'/I_0')_T$ for chamber no. 1 were used in all subsequent calculations.

5·2. *Calculation of correction factors for unscreened sources.* From the corrected experimental results in table 3, obliquity corrections, to be applied to containers of different lengths measured with any one of the three ionization chambers, were calculated in the following manner.

Assume the empirical relation

$$I_x/I_0 = 1 + 3Ax^3 + 5Bx^4 + 7Cx^6, \qquad \text{.....(3)}$$

where I_0 is the measured ionization due to an unscreened point source at the centre of the source-holder at a given distance D cm. from the chamber, I_x is the measured ionization with the same source displaced laterally through a distance x cm., and A, B, C are constants. Uneven powers of x are omitted for reasons of symmetry. Then the effect I' due to a line source extending between x_1 and x_2 will be given by

$$\frac{I'}{I_0} = 1 + A\frac{x_2^3 - x_1^3}{x_2 - x_1} + B\frac{x_2^5 - x_1^5}{x_2 - x_1} + C\frac{x_2^7 - x_1^7}{x_2 - x_1}. \qquad \text{.....(4)}$$

Now the experimental readings were taken of the quantities I_0', I_2', I_4' and I_6' with corresponding values of x as set out below.

Reading	I_0'	I_2'	I_4'	I_6'
Value of x_1	$-0{\cdot}5$	$+1{\cdot}5$	$+3{\cdot}5$	$+5{\cdot}5$ cm.
Value of x_2	$+0{\cdot}5$	$+2{\cdot}5$	$+4{\cdot}5$	$+6{\cdot}5$ cm.

Then using equation (4), eliminating I_0 and solving for A, B and C, we obtain

$$A = -1{\cdot}470\times10^{-1}K_2+2{\cdot}079\times10^{-2}K_4-2{\cdot}163\times10^{-3}K_6, \quad \text{.....(5)}$$

$$B = 9{\cdot}724\times10^{-3}K_2-3{\cdot}265\times10^{-3}K_4+3{\cdot}706\times10^{-4}K_6, \quad \text{.....(6)}$$

$$C = -1{\cdot}868\times10^{-4}K_2+7{\cdot}473\times10^{-5}K_4-1{\cdot}246\times10^{-5}K_6, \quad \text{.....(7)}$$

where

$$K_2 = 1-I_2'/I_0', \quad \text{.....(8)}$$

$$K_4 = 1-I_4'/I_0', \quad \text{.....(9)}$$

$$K_6 = 1-I_6'/I_0'. \quad \text{.....(10)}$$

Thus A, B and C could be calculated directly from the experimental readings by means of equations (5) to (10); and by substituting the values $x_1=-L/2$ and $x_2=L/2$ in equation (4) we obtain the ratio of the measured ionization due to a given amount of radium in a uniform line source of length L cm., to that due to the same amount of radium in a point source, both sources being placed symmetrically in the centre of the source-holder. This ratio is I_0'/I_0, and

$$\frac{I_0'}{I_0} = 1+\frac{A}{4}L^2+\frac{B}{16}L^4+\frac{C}{64}L^6. \quad \text{.....(11)}$$

The obliquity-correction factors are then determined in terms of A, B and C, values of which are known.

The above equations (5) to (11) were used together with the values of $(I_x'/I_0')_T$ from table 3 to calculate values of I_0'/I_0, where I_0 is the measured ionization due to a given amount of radium in an unscreened point source in the centre of a very light source-holder at a distance D cm. from the chamber, and I_0' is the measured ionization due to the same amount of radium in an unscreened line source, of length L cm., placed symmetrically about the centre of the source-holder at a distance D cm. from the same ionization chamber.

The corrections, for ranges of values of L from 0 to 10 cm. and of D from 10 to 60 cm., and for the three ionization chambers, are given in table 4 as values of C_1, which is a percentage addition; that is

$$C_1 = 100\,(I_0/I_0'-1). \quad \text{.....(12)}$$

The figures in the table are uncertain in the last decimal place, the largest corrections probably being correct to within $\pm0{\cdot}2$ per cent while the others have possible errors proportionally smaller. These estimates were deduced from consideration of the average total amount of smoothing applied to the experimental readings in table 3, which was of the order of $\pm0{\cdot}08$ per cent, and also from the

agreement between independent values at distances D of 30 cm. or more for the three chambers, which was of the order of $\pm 0{\cdot}05$ per cent.

Table 3. Values of $(I_x'/I_0')_T$

D (cm.)	x (cm.)	Mean values for chamber no. 1		Mean values for chamber no. 2	Mean values for chamber no. 3
		Wall-thickness 0·3 mm. gold	Wall-thickness 2·1 mm. gold		
10	2	0·9692	0·9669	0·9669	0·9677
	4	0·8829	0·8790	0·8817	0·8848
	6	0·7712	0·7654	0·7700	0·7732
15	2	0·9833	0·9838	0·9841	0·9852
	4	0·9376	0·9388	0·9400	0·9401
	6	0·8714	0·8713	0·8734	0·8740
20	2	0·9897	0·9908	0·9903	0·9908
	4	0·9623	0·9645	0·9631	0·9638
	6	0·9196	0·9220	0·9223	0·9224
30	2	0·9949	0·9960	0·9941	0·9943
	4	0·9807	0·9838	0·9808	0·9806
	6	0·9586	0·9641	0·9613	0·9600
40	2	0·9965	0·9961	—	0·9953
	4	0·9872	0·9892	—	0·9868
	6	0·9731	0·9779	—	0·9742
50	2	0·9974	0·9963	—	0·9963
	4	0·9909	0·9915	—	0·9903
	6	0·9819	0·9841	—	0·9814
60	2	0·9979	0·9966	0·9965	0·9980
	4	0·9944	0·9929	0·9930	0·9932
	6	0·9888	0·9875	0·9887	0·9870

5·3. *Calculation of supplementary obliquity corrections for screened sources.* It must be emphasized that the effects of the container screen and the wooden source-holder were eliminated from the percentage corrections in table 4. Additional corrections to cover these effects were developed by using the preceding equations and approximating, to find that the ratio of the obliquity correction factor for a screened source to that for a similar unscreened source is given approximately by

$$\begin{aligned}1+(\mu_1 s_1+\mu_2 s_2)\{&L^2(3{\cdot}675\times10^{-2}E_2-5{\cdot}198\times10^{-3}E_4+5{\cdot}408\times10^{-4}E_6)\\&-L^4(6{\cdot}078\times10^{-4}E_2-2{\cdot}041\times10^{-4}E_4+2{\cdot}316\times10^{-5}E_6)\\&+L^6(2{\cdot}92\times10^{-6}E_2-1{\cdot}168\times10^{-6}E_4+1{\cdot}946\times10^{-7}E_6)\}\\&=1+(\mu_1 s_1+\mu_2 s_2)F, \qquad\qquad \text{.....(13)}\end{aligned}$$

where

$$E_x=(\sec\theta_x-1)(I_x'/I_0')_T. \qquad \text{.....(14)}$$

This is, of course, another correction factor which must be applied when the source is screened, or when absorption takes place in the source-holder. It is a small correction, and therefore values of $(I_x'/I_0')_T$ from table 3 for any one of the three ionization chambers suffice for the purpose of calculation. It is evident that the term F in equation (13) depends upon L, D, and the quantities $(I_x'/I_0')_T$, and evaluation of this term should enable correction factors to be determined easily for

Table 4. Values of C_1. Percentage obliquity corrections for ionization chambers nos. 1, 2 and 3

Chamber	D (cm.)	$L=1$ cm.	$L=2$ cm.	$L=3$ cm.	$L=4$ cm.
1	10	0·07	0·29	0·64	1·14
2	10	0·07	0·29	0·64	1·14
3	10	0·07	0·28	0·61	1·08
1	15	0·03	0·14	0·31	0·54
2	15	0·03	0·13	0·30	0·53
3	15	0·03	0·13	0·28	0·51
1	20	0·02	0·08	0·19	0·33
2	20	0·02	0·08	0·19	0·32
3	20	0·02	0·08	0·18	0·32
All	30	0·01	0·05	0·11	0·18
All	40	0·01	0·04	0·09	0·14
All	50	0·01	0·03	0·08	0·12
All	60	0·01	0·03	0·07	0·10

Chamber	D (cm.)	$L=5$ cm.	$L=6$ cm.	$L=8$ cm.	$L=10$ cm.
1	10	1·77	2·53	4·4	6·8
2	10	1·76	2·51	4·3	6·6
3	10	1·69	2·41	4·1	6·5
1	15	0·85	1·21	2·14	3·3
2	15	0·83	1·19	2·10	3·2
3	15	0·80	1·15	2·05	3·2
1	20	0·51	0·73	1·28	1·96
2	20	0·51	0·73	1·28	1·96
3	20	0·49	0·72	1·27	1·95
All	30	0·28	0·40	0·66	0·98
All	40	0·22	0·30	0·49	0·70
All	50	0·18	0·24	0·39	0·54
All	60	0·15	0·20	0·30	0·40

various values of $\mu_1 s_1$ and $\mu_2 s_2$. Values of F were calculated and are given to the required degree of accuracy in Table 5 for different distances D between the centres of the source and ionization chamber, and for different lengths L of source. The additional percentage correction C_2 to be added to the measured content of a screened container is, then, by equation (13), given by

$$C_2 = 100F(\mu_1 s_1 + \mu_2 s_2). \qquad \text{.....(15)}$$

This correction is quite appreciable under certain experimental conditions.

§6. DISCUSSION OF CORRECTION FOR OBLIQUITY

A consideration of the results described above shows how the effects of obliquity on the measurement of radioactive sources of finite length are influenced by such factors as the shape, size and lead-wall-thickness of the ionization chamber, and by the presence of the source-holder and container wall. This renders inaccurate the use of the inverse square law for small or moderate distances between the source and measuring instrument. With the given shape of chamber, the lead-wall-

thickness has a negligible effect for source-to-chamber distances of 30 cm. or more. For smaller distances, increasing the lead-wall-thickness reduces the obliquity corrections, as would be expected from geometrical considerations.

It has been shown that a supplementary correction C_2 is necessary to allow for the oblique path of the measured radiation from the outer portions of the source in passing through the container screen. This correction may amount to as much as 0·7 per cent for a container 10 cm. in length, of wall-thickness 2 mm. of platinum, and placed 10 cm. from the centre of the chamber; for distances greater than 30 cm., however, it is practically negligible, being less than 0·05 per cent.

Table 5. Value of term F

D (cm.)	$L=1{\cdot}5$ cm.	$L=2$ cm.	$L=3$ cm.	$L=4$ cm.
10	0·0008	0·0017	0·0039	0·0066
15	—	0·0006	0·0015	0·0025
20	—	0·0005	0·0009	0·0016
30	—	—	0·0004	0·0007
40	—	—	—	—
50	—	—	—	—
60	—	—	—	—

D (cm.)	$L=5$ cm.	$L=6$ cm.	$L=8$ cm.	$L=10$ cm.
10	0·0101	0·0143	0·0248	0·0362
15	0·0040	0·0060	0·0111	0·0169
20	0·0025	0·0036	0·0065	0·0099
30	0·0010	0·0014	0·0027	0·0041
40	0·0005	0·0008	0·0016	0·0025
50	—	0·0007	0·0011	0·0017
60	—	0·0006	0·0009	0·0012

Backhurst[27] has calculated obliquity corrections for containers measured with a cylindrical ionization chamber with a plane circular front covered by a lead screen. His calculations take into account the front lead screen, but involve certain approximations and neglect such factors as the wall-thickness of the source, and the effect of radiation incident on the sides and back of the chamber. The latter probably has an appreciable effect, and it is to be expected that the calculated corrections would be too great for this reason. Backhurst compared some of his corrections with experiment and did in fact find them too great; to account for the difference he then assumed that the calculated corrections were accurate, and that radiation scattered from the radium-source-holder and surrounding objects constituted about 20 per cent of the total radiation entering his ionization chamber. This assumption does not appear to agree with the results described in § 3 above.

§ 7. CORRECTIONS FOR WALL ABSORPTION

The magnitude of the correction for absorption in the wall of a radium-container is, like that of the obliquity correction, determined by a number of geometrical and experimental conditions, of which the following are the most important: (*a*) the wall-material, wall-thickness and diameter of the container; (*b*) the fraction of the

inner volume that is filled with radioactive salt—the percentage correction for a partially filled container is somewhat greater than that for a completely filled container[38]; (*c*) the angle subtended by the ionization chamber at the centre of the source; (*d*) the shape and size of the ionization chamber; (*e*) the nature of the lining and the lead-wall-thickness of the ionization chamber.

In addition, a correction may often be necessary for the absorption and scattering of radiation traversing different thicknesses of the source-holder before entering the measuring instrument.

The scope of the present work has been limited to platinum, gold and silver containers, of wall-thickness not exceeding 2 mm., measured with the three cylindrical ionization chambers already specified.

§ 8. METAL SHEATHS USED FOR MEASUREMENTS

The elementary radioactive sources used consisted of pieces of metal capillary tubing, of length 2 cm. and internal diameter 0·15 mm., filled with radon in equilibrium with its decay products. Composite sources were formed by using sets of cylindrical metal sheaths constructed from platinum, gold and silver with a purity of better than 99·5 per cent. Measurements were made of the density of each sheath; this was found to be constant for all the sheaths of each set and for solid samples of the same metals. The densities (g./cm.^{3}) were as follows: platinum, 21·3; gold, 19·2; silver, 10·44. These values are slightly low, probably owing to the somewhat porous nature of the metals, which were purchased in the form of cast rod. The internal diameters of the sheaths were slightly in excess of the external diameter of the elementary source, which could thus be slid into each sheath in turn, and the outer sheaths were made of total length 3 or 4 cm. so that they might provide sufficient overlap at each end to ensure that all of the measured radiation should pass through a uniform container wall.

For measurements with sheaths of platinum and gold, which are of approximately the same density and atomic number, gold capillary tubing was used for the elementary sources. With silver sheaths, however, silver capillary tubing was used, in order to avoid possible errors in extrapolating the results to zero wall-thickness. Such errors could arise from differences in the quality of the gamma-ray beams emerging from gold and silver elementary sources.

Since the wall-absorption correction for a thickness of 1 mm. of platinum is of the order of 10 per cent, it was desirable to determine the wall-thicknesses of the sheaths to within about $\pm 0{\cdot}005$ mm. It is difficult to attain this degree of accuracy by measuring the internal diameter at the ends only, since the hole may vary in diameter or the ends may be irregular.

Measurement of the internal diameter at points along the length of each sheath was finally effected by the following method. Steel sewing needles of various sizes were obtained, and two were selected to slide into the sheaths giving as close a fit as possible. Two needles were necessary since the holes in some of the sheaths were somewhat smaller than those in the others. The surface of each selected needle was

etched deeply, a length of only about 5 mm. being left untouched in the centre, and the etched portion of the eyelet side was then cut off. Measurements of the angle of tilt of one of these needles when the unetched portion was inside a sheath then enabled the internal diameter of the latter to be found as follows. Let D be the internal diameter of the sheath, d the external diameter of the unetched part of the needle, l the length of the unetched part of the needle, L the length of the etched part of the needle on the point side, 2θ the angle of tilt of the needle, and $2y$ the displacement of the point of the needle for this angle. Then it is easily shown that

$$D = l \sin\theta + d\cos\theta \quad \text{and} \quad y = (L + \tfrac{1}{2}l)\sin\theta, \qquad \ldots\ldots(16)$$

whence D may be found when d, l, L and y are known.

The value of d was found with a micrometer, while l and L were determined with a travelling microscope. Average values of $2y$ were found with the aid of an optical projection system giving a magnification of 50·09. The distance $50{\cdot}09 \times 2y$ was of the order of 3 cm. and could be measured to about 1 mm., which corresponds to an error in d of $\pm 0{\cdot}002$ mm. Readings were taken at three positions along the length of each sheath in each of two planes at right angles. The calculated average values of the internal radii, which are given in table 6, are considered to be of the desired order of accuracy of $\pm 0{\cdot}005$ mm. Periodical weighing of the sheaths showed that a negligible amount of wearing took place during use, and the internal diameters remained practically constant. In table 6 are included also values of the total lengths, external radii and wall-thicknesses of the sheaths and elementary sources.

Table 6. Metal sheaths used in absorption measurements

Sheath	Length (mm.)	External radius (mm.)	Internal radius (mm.)	Wall-thickness (mm.)
Pt *A*	20·0	0·408	0·071	0·336
Pt *B*	30·9	0·982	0·464	0·519
Pt *C*	30·7	1·131	0·463	0·668
Pt *D*	30·6	1·994	0·539	1·455
Pt *E*	30·1	2·379	0·546	1·834
Au *A*	20·0	0·408	0·071	0·336
Au *B*	31·2	0·963	0·469	0·494
Au *C*	41·4	1·406	0·457	0·949
Au *D*	41·2	1·765	0·460	1·305
Au *E*	30·0	2·073	0·539	1·535
Au *F*	30·8	2·378	0·536	1·842
Ag *A*	20·0	0·414	0·075	0·338
Ag *B*	31·1	0·992	0·463	0·530
Ag *C*	31·6	1·378	0·458	0·920
Ag *D*	31·4	1·771	0·466	1·305
Ag *E*	31·2	2·081	0·527	1·554
Ag *F*	30·5	2·381	0·533	1·848

§ 9. CALCULATION OF EFFECTIVE WALL THICKNESS

9·1. *Effect of size of ionization chamber.* Since the ionization chamber was of finite width and subtended an appreciable angle at the centre of the source, the magnitude of which depended upon the source-to-chamber distance, a correction

was applied for the obliquity of the path of the measured radiation traversing the container wall. Owing to the horizontal position of the cylindrical sources, the height of the chamber has a negligible influence on this effect, which applies even to sources of small active length and is distinct from that due to the obliquity of the path through the container wall of radiation from the outer portions of a source of finite active length, which was discussed in § 5 of this paper.

The correction was applied by finding approximately the average wall-thickness traversed by rays from a short source; thus in figure 4 let D be the distance between the centre P of the source and the centre O of the chamber; r the internal radius of the chamber, s the wall-thickness of the source, and 2θ the angle subtended by the

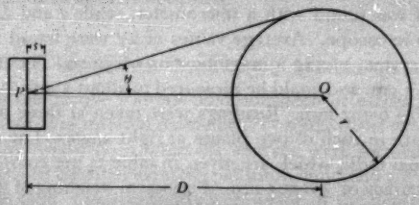

Figure 4.

chamber at the centre of the source. Then the average wall-thickness s' traversed by rays entering the chamber from P is given by

$$s' = \frac{1}{\theta}\int_0^{\theta} s \sec\phi \, d\phi;$$

therefore

$$\frac{s'}{s} = \frac{1}{\theta}\log_e \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right). \qquad \text{.....(17)}$$

For the ionization chambers used $r = 4{\cdot}75$ cm., which gives the following values for s'/s:

D (cm.)	10	15	20	30	40	60
s'/s	1·0435	1·0180	1·0097	1·0042	1·0024	1·0011

From these figures it is evident that the effect under consideration was appreciable for moderate values of D, varying between 0·44 per cent and 0·01 per cent for values of D of 10 and 60 cm. respectively, for the case of a container screened with a platinum sheath 1 mm. thick.

9·2. *Effect of clearance between sheath and elementary source.* The experimental measurements were made with composite sources, each of which consisted of an elementary source surrounded by one solid metal sheath of known material and wall-thickness. Since the external diameter of the inside source was always less than the internal diameter of the sheath, the inside source lay in a slightly eccentric position, causing the measured beam to traverse a thickness of metal slightly greater than the actual wall-thickness of the sheath. This necessitated corrections to the

wall-thicknesses of the order of 0·001 to 0·014 mm. For this purpose the following approximate relation was used:

$$w=\sqrt{\{b^2-(a-a_1)^2\}}-\sqrt{\{a^2-(a-a_1)^2\}}, \qquad \text{.....(18)}$$

where w is the corrected wall-thickness, b the external radius of the sheath, a the internal radius of the sheath, and a_1 the external radius of the elementary source.

In table 7 are given the total wall-thicknesses of the composite sources used in the experiments, and also the effective wall-thicknesses calculated by means of equations (17) and (18), for the appropriate distances from the centre of the ionization chamber.

§ 10. CORRECTION FOR WOODEN SOURCE-HOLDER

In order to correct the experimental readings for absorption and scattering of the measured gamma-rays in passing through the V-shaped wooden holder in which the sources were supported, it was considered advisable to determine experimentally the apparent absorption coefficient of gamma-rays in wood. For this purpose wooden blocks 5 cm. long, 2·5 cm. high, and of different thicknesses were prepared from cedar of density 0·40 g./cm.^{3}, which wood was also used for the source-holder.

Readings were taken with each of the three ionization chambers, the absorbing blocks being located as close as possible to the source, which was a 10-mg. radium needle of active length 2 cm. and wall-thickness 0·5 mm. of platinum. From the mean readings the following apparent absorption coefficients were calculated for wood-thicknesses not exceeding 6 mm.: for chamber no. 1, $\mu=0{\cdot}0135$ cm.$^{-1}$; for chamber no. 2, $\mu=0{\cdot}0156$ cm.$^{-1}$; for chamber no. 3, $\mu=0{\cdot}0139$ cm.$^{-1}$. The mean value $\mu=0{\cdot}0143$ cm.$^{-1}$ was used in calculating correction factors for the experimental readings with metal sheaths of varying diameter. These factors were as follows:

Composite source	Wood correction factor
Pt A, Au A and Ag A	1·0075
Pt $(A+B)$, Au $(A+B)$, and Ag $(A+B)$	1·0054
Pt $(A+C)$	1·0047
All other sources	1·0000

As a check on these factors, a separate measurement was made of the correction for the elementary sources (Pt A, Au A and Ag A) by using a radon source made of silver capillary tubing and replacing the wooden source-holder by a light one made of paper. The correction factor found by this means was 1·0080, which agrees with the value given to 0·05 per cent.

§ 11. EXPERIMENTAL READINGS WITH METAL SHEATHS

For the determination of the effect of absorption in the cylindrical metal container screens, series of readings were taken with each set of sheaths at two or three different distances from each of the three ionization chambers. Readings were

Table 7. Total wall-thickness of composite sources

Platinum				Gold						Silver			
Source	True wall-thickness (mm.)	Apparent wall-thickness (mm.)		Source	True wall-thickness (mm.)	Apparent wall-thickness (mm.)				Source	True wall-thickness (mm.)	Apparent wall-thickness (mm.)	
		$D=20$ cm.	$D=30$ cm.			$D=10$ cm.	$D=20$ cm.	$D=30$ cm.	$D=40$ cm.			$D=20$ cm.	$D=30$ cm.
A	0·336	0·339	0·337	A	0·336	0·350	0·339	0·337	0·337	A	0·338	0·341	0·339
$A+B$	0·855	0·864	0·859	$A+B$	0·830	0·867	0·840	0·835	0·834	$A+B$	0·868	0·877	0·872
$A+C$	1·004	1·016	1·010	$A+C$	1·285	1·341	1·298	1·291	1·289	$A+C$	1·258	1·271	1·264
$A+D$	1·791	1·820	1·810	$A+D$	1·641	1·714	1·659	1·649	1·647	$A+D$	1·643	1·662	1·652
$A+E$	2·170	2·205	2·192	$A+E$	1·871	1·964	1·901	1·890	1·888	$A+E$	1·892	1·919	1·908
				$A+F$	2·178	2·284	2·210	2·198	2·196	$A+F$	2·186	2·217	2·204

Table 8. Mean experimental readings of relative ionization with cylindrical metal sheaths

Source	Chamber no. 1				Chamber no. 2		Chamber no. 3	
	$D=10$ cm.	$D=20$ cm.	$D=30$ cm.	$D=40$ cm.	$D=20$ cm.	$D=30$ cm.	$D=20$ cm.	$D=30$ cm.
Pt A	—	1·0000	1·0000	—	1·0000	1·0000	1·0000	1·0000
$A+B$	—	0·9462	0·9447	—	0·9525	0·9508	0·9556	0·9548
$A+C$	—	0·9302	0·9289	—	0·9369	0·9385	0·9414	0·9409
$A+D$	—	0·8552	0·8559	—	0·8692	0·8701	0·8736	0·8769
$A+E$	—	0·8241	0·8244	—	0·8406	0·8370	0·8436	0·8490
Au A	1·0000	1·0000	—	1·0000	1·0000	1·0000	1·0000	1·0000
$A+B$	0·9513	0·9552	—	0·9527	0·9612	0·9538	0·9614	0·9606
$A+C$	0·9054	0·9060	—	0·9062	0·9184	0·9191	0·9203	0·9238
$A+D$	0·8738	0·8774	—	0·8791	0·8904	0·8920	0·8930	0·8987
$A+E$	0·8550	0·8550	—	0·8557	0·8721	0·8727	0·8795	0·8787
$A+F$	0·8293	0·8326	—	0·8359	0·8521	0·8491	0·8568	0·8539
Ag A	—	1·0000	1·0000	—	1·0000	1·0000	1·0000	1·0000
$A+B$	—	0·9847	0·9839	—	0·9818	0·9829	0·9820	0·9823
$A+C$	—	0·9671	0·9707	—	0·9644	0·9640	0·9637	0·9645
$A+D$	—	0·9562	0·9586	—	0·9557	0·9530	0·9510	0·9552
$A+E$	—	0·9493	0·9505	—	0·9464	0·9452	0·9439	0·9407
$A+F$	—	0·9418	0·9434	—	0·9386	0·9369	0·9324	0·9308

taken in varying orders, to eliminate effects due to the decay of the radon and to the zero-drift of the valve amplifier, and with the elementary radon source alternately unsheathed and inside each member of the set of sheaths.

In each case about 20 readings were taken, and the mean values were corrected where necessary for (*a*) absorption in the wooden holder (see § 10 above), and (*b*) the effect of the finite length of the source (see § 5). In table 8 are given the mean corrected experimental values of the ratios

$$\frac{I_{A+B}}{I_A}, \quad \frac{I_{A+C}}{I_A},$$

and so on, where I_A is the measured ionization from the elementary radon source A, I_{A+B} is the measured ionization from the elementary radon source A inside sheath B, and so on.

From each of the independent series of readings in table 8, absorption correction factors corresponding to *apparent* wall-thicknesses of 0·5, 1, 1·5 and 2·0 mm. were calculated algebraically. For this purpose the values of the apparent wall-thicknesses of the metal sheaths in table 7 were used, and the figures in table 8 were smoothed to the average extent of ±0·09 per cent and extrapolated to zero wall-thickness to allow for absorption in the wall of the elementary source A. Table 9 shows the results of these calculations, expressed as percentage corrections to be added to the measured content of a container.

It will be seen that the calculated corrections have mean departures from the mean values of about ±0·2 per cent, and that, for the results with gold screens measured at widely differing distances from ionization chamber no. 1, this figure is ±0·05 per cent. This agreement is considered to justify the use of the apparent wall-thickness of the source rather than the true wall-thickness, in the manner set out above in § 9.

§ 12. DERIVATION OF CORRECTIONS FOR CONTAINERS OF ZERO INTERNAL DIAMETER

The figures in table 9 apply only to radon sources of internal diameter 0·15 mm. It was necessary, therefore, to modify them slightly to allow for the finite size of the inside hole, and for the fact that the source was empty so that most of the active deposit was on the inside metal surface, rather than distributed throughout the internal volume as in a radium-container full of radioactive salt.

The mean wall-thickness w_1 traversed by a parallel beam of gamma-rays emerging from an empty container of internal radius a and external radius b is, assuming the active deposit to be on the inside walls only, given by

$$w_1 = \frac{a}{\pi} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} (\sqrt{\{b^2/a^2 - \sin^2\theta\}} - \cos\theta)\, d\theta. \qquad \text{.....(19)}$$

Similarly, for a full container with active deposite distributed uniformly throughout the internal volume, the mean thickness w_2 is given by

$$w_2 = \frac{2a}{\pi} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos^2\theta\, (\sqrt{\{b^2/a^2 - \sin^2\theta\}} - \cos\theta)\, d\theta. \qquad \text{.....(20)}$$

Table 9. Percentage absorption corrections calculated from experimental readings

Metal	Wall-thickness (mm.)	Chamber no. 1					Chamber no. 2			Chamber no. 3		
		$D = 10$ cm.	$D = 20$ cm.	$D = 30$ cm.	$D = 40$ cm.	Mean	$D = 20$ cm.	$D = 30$ cm.	Mean	$D = 20$ cm.	$D = 30$ cm.	Mean
Pt	0·5	—	5·26	5·83	—	5·54	4·82	4·93	4·88	4·23	4·45	4·34
	1·0	—	11·04	11·73	—	11·38	9·90	10·05	9·98	8·98	9·26	9·12
	1·5	—	17·14	17·70	—	17·42	15·20	15·39	15·30	14·15	14·26	14·20
	2·0	—	23·34	23·84	—	23·59	20·68	21·03	20·85	19·61	19·34	19·48
Au	0·5	4·65	4·52	—	4·52	4·56	4·23	4·84	4·54	3·80	4·15	3·98
	1·0	10·00	9·96	—	9·91	9·96	8·97	9·64	9·30	8·37	8·51	8·44
	1·5	15·68	15·77	—	15·86	15·77	13·94	14·47	14·20	13·19	13·18	13·18
	2·0	21·34	21·46	—	21·29	21·36	18·88	19·49	19·18	17·78	18·21	18·00
Ag	0·5	—	2·15	1·48	—	1·82	1·98	1·75	1·86	1·88	1·89	1·88
	1·0	—	4·02	3·15	—	3·58	3·88	3·66	3·77	3·79	3·78	3·78
	1·5	—	5·66	4·83	—	5·24	5·69	5·58	5·64	5·76	5·75	5·76
	2·0	—	7·17	6·47	—	6·82	7·34	7·33	7·34	7·78	7·79	7·78

Table 10. Values of C_0, the percentage corrections for cylindrical containers of zero internal diameter

Apparent wall-thickness (mm.)	Platinum			Gold			Silver		
	Ionization chamber			Ionization chamber			Ionization chamber		
	No. 1	No. 2	No. 3	No. 1	No. 2	No. 3	No. 1	No. 2	No. 3
0·1	0·98	0·87	0·75	0·79	0·73	0·70	0·34	0·34	0·34
0·2	2·01	1·78	1·54	1·63	1·52	1·44	0·68	0·70	0·70
0·3	3·08	2·72	2·39	2·53	2·36	2·21	1·03	1·07	1·07
0·4	4·16	3·68	3·24	3·48	3·23	3·01	1·38	1·44	1·45
0·5	5·28	4·64	4·12	4·46	4·12	3·83	1·73	1·81	1·83
0·6	6·41	5·64	5·04	5·50	5·06	4·70	2·08	2·19	2·21
0·7	7·55	6·63	5·96	6·55	6·01	5·56	2·43	2·57	2·59
0·8	8·72	7·65	6·92	7·61	6·97	6·46	2·78	2·95	2·97
0·9	9·89	8·68	7·87	8·71	7·95	7·37	3·13	3·34	3·35
1·0	11·07	9·72	8·87	9·80	8·93	8·27	3·48	3·72	3·73
1·1	12·3	10·8	9·9	10·9	9·9	9·2	3·8	4·1	4·1
1·2	13·5	11·8	10·9	12·0	10·9	10·1	4·2	4·5	4·5
1·3	14·7	12·9	11·9	13·2	11·9	11·0	4·5	4·9	4·9
1·4	15·9	13·9	12·9	14·3	12·9	12·0	4·8	5·2	5·3
1·5	17·1	15·0	13·9	15·4	13·9	12·9	5·2	5·6	5·7
1·6	18·3	16·1	15·0	16·5	14·9	13·8	5·5	6·0	6·1
1·7	19·6	17·2	16·0	17·7	15·8	14·8	5·8	6·3	6·5
1·8	20·8	18·3	17·1	18·8	16·8	15·7	6·1	6·6	6·9
1·9	22·1	19·4	18·1	19·9	17·8	16·7	6·4	7·0	7·3
2·0	23·4	20·6	19·2	21·0	18·8	17·6	6·7	7·3	7·7

It is found that w_1 is slightly greater than w_2. The percentage correction for a full source is approximately w_2/w_1 times that for an empty source; this approximation was considered to be sufficiently accurate for the adjustment of the corrections in table 9, since the required alterations were only of the order of from 0·02 to 0·15 per cent.

In the same manner, the corrections were modified to obtain values applicable to radioactive containers of zero internal diameter by means of the formula of Owen and Naylor[28, 30], which is considered in greater detail in § 13 below. This adjustment also did not exceed 0·15 per cent.

In table 10 are given values of C_0, calculated from table 9 in the manner indicated, where C_0 is the percentage correction that must be added to the measured content of a radium-container of given apparent wall-thickness and zero internal diameter, in order to obtain the true content. These corrections are considered to be correct to within $\pm 0{\cdot}2$ per cent for the larger wall-thicknesses and to within $\pm 0{\cdot}1$ per cent for smaller thicknesses. The values of C_0 are shown in graphical form in figure 5, together with the unsmoothed mean experimental points which will be seen to lie fairly closely on the curves.

§ 13. MODIFICATION OF CORRECTIONS FOR CONTAINER OF FINITE INTERNAL DIAMETER

Owen and Naylor[30] deduced from geometrical considerations that the measured ionization due to a parallel beam of rays emerging from a cylindrical tube, of internal radius a and external radius b, would be reduced by the presence of the tube in the ratio given by the equation

$$\frac{I}{I_0}=\frac{2}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\cos^2\theta e^{-\mu a\left(\sqrt{(b^2/a^2-\sin^2\theta)}-\cos\theta\right)}.d\theta, \qquad \text{.....(21)}$$

where μ is the absorption coefficient of the gamma-rays in the wall-material of the tube.

Although the effect of secondary radiation from the absorbing tube renders it inadvisable to rely completely on this formula, Kaye, Aston and Perry[28] have shown that the latter may be used in deriving the absorption correction for a tube of given diameter from the experimentally determined correction for a tube of different diameter but the same wall-thickness.

Thus if c_e is the experimental correction for the first tube and c_f the calculated correction for the first tube using the formula (21), and if c_e' and c_f' are the corresponding quantities for the second tube, then

$$\frac{c_e}{c_e'}=\frac{c_f}{c_f'}. \qquad \text{.....(22)}$$

Furthermore, calculations show that the ratio c_f/c_f' remains constant to within the required limits for given values of a and b, and values of μ varying between 1·0 and 0·3 cm.$^{-1}$. This method of calculation was therefore used in obtaining the absorption correction C_0 for zero internal diameter from the experimental correc-

tions determined with sources of 0·15 mm. bore. It may also be used within limits to derive the corrections for containers of given internal diameter from the values of C_0 in table 10. For use in this respect table 11 is included, giving the calculated values of k for containers of different internal diameters and wall-thicknesses, where k is the ratio of the percentage correction for a container of given internal diameter

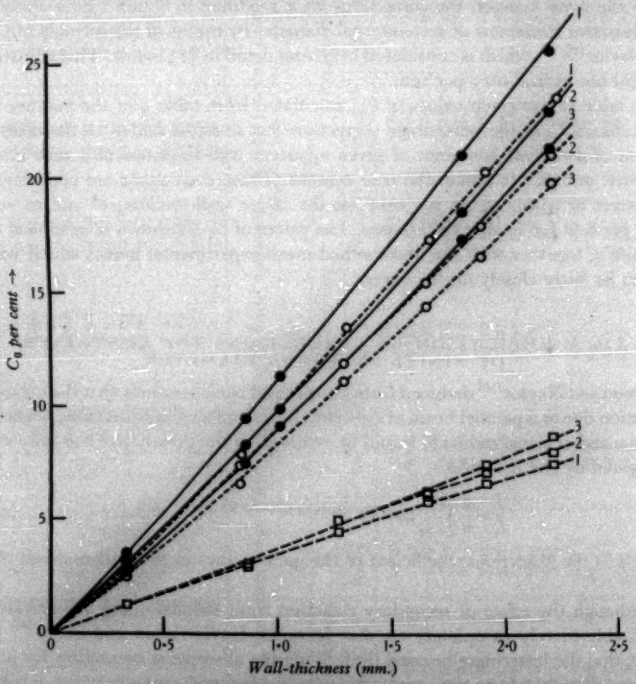

Figure 5. Wall-absorption correction C_0 for cylindrical containers of zero internal diameter when cylindrical ionization chambers nos. 1, 2 and 3 are used

—●—, platinum; ---○---, gold; ---□---, silver.

and wall-thickness to the percentage correction C_0 for a container of zero internal diameter and the same wall-thickness. Then the required correction C_s is given by

$$C_s = kC_0. \qquad \text{.....(23)}$$

The results obtained by Kaye, Aston and Perry[28] show that the above general method of allowing for the diameter of the container is satisfactory over the range of internal diameters from 0·8 to 4·9 mm., while a test measurement by the writer covers the range 0·15 to 1·8 mm. The latter test was carried out with a gold sheath of

Table 11. Values of k

Wall-thickness (mm.)	Internal diameter of radium-containers (mm.)					
	0·15	0·5	1·0	1·5	2·0	2·5
0·1	1·036	1·092	1·135	1·159	1·180	1·196
0·2	1·032	1·083	1·120	1·147	1·166	1·181
0·3	1·029	1·074	1·112	1·134	1·151	1·165
0·4	1·025	1·066	1·101	1·121	1·136	1·152
0·5	1·022	1·059	1·091	1·111	1·126	1·140
0·6	1·019	1·053	1·082	1·102	1·116	1·130
0·7	1·017	1·047	1·075	1·094	1·108	1·120
0·8	1·015	1·041	1·068	1·086	1·100	1·112
0·9	1·013	1·037	1·063	1·080	1·094	1·104
1·0	1·012	1·034	1·058	1·075	1·088	1·098
1·1	1·011	1·031	1·054	1·070	1·083	1·093
1·2	1·010	1·028	1·050	1·066	1·078	1·090
1·3	1·009	1·026	1·047	1·063	1·075	1·086
1·4	1·009	1·025	1·044	1·060	1·072	1·083
1·5	1·008	1·023	1·042	1·057	1·069	1·080
1·6	1·008	1·022	1·040	1·055	1·067	1·077
1·7	1·007	1·021	1·039	1·053	1·064	1·075
1·8	1·007	1·020	1·038	1·051	1·062	1·073
1·9	1·006	1·020	1·037	1·049	1·060	1·071
2·0	1·006	1·019	1·035	1·047	1·058	1·069

Wall-thickness (mm.)	Internal diameter of radium-containers (mm.)					
	3·0	4·0	5·0	6·0	7·0	8·0
0·1	1·210	1·235	1·245	1·253	1·255	1·256
0·2	1·195	1·213	1·226	1·235	1·240	1·241
0·3	1·179	1·197	1·211	1·222	1·226	1·227
0·4	1·165	1·182	1·199	1·208	1·211	1·215
0·5	1·152	1·171	1·186	1·196	1·201	1·205
0·6	1·140	1·160	1·175	1·185	1·192	1·197
0·7	1·129	1·151	1·165	1·176	1·185	1·189
0·8	1·120	1·142	1·158	1·168	1·178	1·182
0·9	1·113	1·134	1·151	1·162	1·171	1·176
1·0	1·108	1·128	1·144	1·156	1·165	1·171
1·1	1·104	1·123	1·139	1·151	1·160	1·166
1·2	1·100	1·118	1·135	1·147	1·156	1·161
1·3	1·096	1·115	1·131	1·143	1·152	1·157
1·4	1·093	1·111	1·127	1·139	1·149	1·153
1·5	1·090	1·108	1·124	1·136	1·145	1·150
1·6	1·087	1·105	1·120	1·132	1·141	1·146
1·7	1·084	1·102	1·117	1·128	1·139	1·144
1·8	1·082	1·099	1·113	1·125	1·136	1·141
1·9	1·080	1·096	1·110	1·122	1·133	1·138
2·0	1·078	1·094	1·107	1·120	1·130	1·136

length 3 cm., external diameter 4·729 mm., and internal diameter 1·777 mm. in conjunction with a soda-glass source of length 1·6 cm., external diameter 1·45 mm., and internal diameter 0·818 mm., filled with radon. The inside of the glass source was tightly packed with glass wool in order to approximate to a full radium-container. The experimental correction for the gold sheath, measured with ionization chamber no. 1, was found to be 13·97 per cent, while the correction calculated on the lines

indicated above from table 10 was 14·07 per cent. This difference was within the limits of experimental error.

An additional test was carried out to determine whether the influence of secondary radiation from the outer parts of a cylinder of silver varied with the distance between these metal parts and a central radioactive part; for this a silver sheath of apparent wall-thickness 1·63 mm. and internal diameter 8·8 mm. was used, with a radon source of silver capillary tubing supported in the centre. The measured percentage corrections were found to be 5·44, 6·00 and 6·17 per cent for ionization chambers 1, 2 and 3 respectively, while the corresponding corrections calculated from table 10 were 5·57, 6·05 and 6·23 per cent. The differences between these results will be seen to be of the order of 0·08 per cent, indicating that secondary radiation from the container walls should have little effect on the values of k in equation (23), even in the case of silver sheaths measured with ionization chamber no. 1.

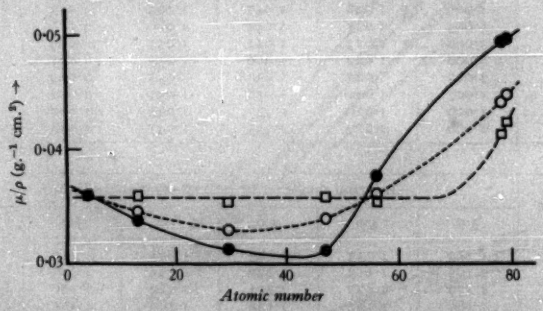

Figure 6. Variation of μ/ρ with atomic number for ionization chambers nos. 1, 2 and 3.
——●——, chamber no. 1; - - - -○- - - -, chamber no. 2; – – –□– – –, chamber no. 3.

§ 14. VARIATION OF MASS-ABSORPTION COEFFICIENT

It was considered of interest to follow the variation of the mass-absorption coefficient for gamma-rays with the atomic number of the absorbing material and with the thickness of the lead filter on the cylindrical ionization chamber, so further readings were taken with some sheaths, similar in shape to the thickest gold sheath, but made of aluminium, brass and an alloy of 10 parts by weight of silver and 7 parts of gold. The measured values of the mass absorption coefficient μ/ρ are given in table 12 for sheaths of thickness 2 mm.

The curves in figure 6, where μ/ρ is plotted against the atomic number of the absorbing material, show that as the thickness of the lead filter is increased, μ/ρ tends to a constant value for materials of lower atomic number, but that photoelectric and nuclear absorption play an important role for heavier elements such as gold and platinum. The increase in μ/ρ with increasing thickness of lead filter for the lighter materials shows that secondary radiation[31], emitted from parts of the absorbing sheath not in the direct measured beam, adds considerably to the measured ionization.

Table 12. Measured mass absorption coefficients

Material	Atomic number	Density (g./cm.^{3})	Mass absorption coefficient, μ/ρ		
			Chamber no. 1	Chamber no. 2	Chamber no. 3
Wood	4	0·40	0·036	0·036	0·036
Aluminium	13	2·70	0·0338	0·0346	0·0360
Brass	29·5	8·5	0·0313	0·0330	0·0354
Silver	47	10·44	0·0312	0·0340	0·0359
Silver-gold alloy	55·9	12·94	0·0377	0·0361	0·0355
Platinum	78	21·3	0·0495	0·0441	0·0413
Gold	79	19·2	0·0497	0·0448	0·0424

From the results given in table 12 and figure 6 it should be possible to deduce, at least approximately, the percentage corrections to allow for absorption in cylindrical containers of wall-materials other than those covered by table 10.

§ 15. DISCUSSION OF RESULTS

The absorption results described in this paper show a general agreement with those of Kaye, Aston and Perry[28] for platinum screens, but it can be seen that the curved front of the ionization chamber used here causes the absorption curves to assume a shape for the smaller thicknesses of the absorber slightly different from that which they have when the ionization chamber has a plane front. This effect is to be expected from geometrical considerations.

It must be emphasized that the results given may apply only to a cylindrical ionization chamber lined with brass, and errors may arise[32, 33] if they are applied to measurements made with a chamber of different lining, of different shape, or of widely different volume.

Finally, it has been shown that, to obtain the true content of a radium-container from the measured content, the latter must be multiplied by a number of correction factors according to the following relation:

$$\frac{I_0}{I}=\left(1+\frac{C_1}{100}\right)\left(1+\frac{C_2}{100}\right)\left(1+\frac{C_3}{100}\right)\left(1+\frac{C_4}{100}\right)\left(1+\frac{C_5}{100}\right), \quad \ldots\ldots(24)$$

where C_1 is the percentage correction for obliquity (§ 5·2), C_2 is the supplementary percentage correction for obliquity (§ 5·3), C_3 is the percentage correction for absorption in the apparent wall of the container (§ 13), C_4 is the percentage correction for absorption in the source-holder (§ 10), and C_5 is the percentage correction for absorption in the radioactive salt itself[34, 35, 36].

The relative accuracy of these experimentally determined factors was tested in the following manner. A laboratory substandard 20-mg. radium tube (of screenage 1·0 mm. platinum) was calibrated in terms of the Australian national substandard tube (of screenage 0·27 mm. glass) under different experimental conditions, the results being given in table 13. It is seen that the corrections produce concordant results over a wide variety of experimental conditions.

Evidence of the absolute accuracy of the corrections was obtained by measuring

a particular radium-container that has recently been calibrated at the National Physical Laboratory. This involved at least six independent measurements. The National Physical Laboratory calibration involved the comparison of the container with at least one working substandard which had been calibrated in terms of the British national substandard. In this case, the corrections determined by the National Physical Laboratory for their particular experimental conditions would be used. The measurement in the Commonwealth Laboratory involved a similar set of comparisons based on the Australian national substandard, using the corrections for the experimental conditions described in this paper. The results of four independent experiments gave for the true radium-content 10·34, 10·41, 10·46 and 10·40 mg., with a mean of 10·40, and a mean departure of ±0·03: the content as certified by the National Physical Laboratory was 10·40. Although this absolute agreement is rather fortuitous, it can be claimed that the corrections deduced in this paper are sufficiently accurate for the present purposes.

Table 13

Ionization chamber no.	Source-to-chamber distance (cm.)	National tube correction factor	Laboratory tube correction factor	Laboratory tube corrected content (mg.)
1	30	1·0050	1·1232	20·44
1	40	1·0048	1·1229	20·24
2	30	1·0048	1·1084	20·38
2	40	1·0046	1·1082	20·35
3	20	1·0053	1·1003	20·27
3	30	1·0046	1·0994	20·28

Mean 20·33 ± 0·06

By experience gained in this work, the measuring instrument described has been proved to be eminently suitable for ionization measurements, when a large number of readings have to be taken speedily and with small exposure of the operator. A fair degree of reproducibility (of the order of ±0·3 per cent) is obtained, especially with moderate source-to-chamber distances (15–40 cm.), with which large ionization currents are produced. On the other hand, the high initial cost of a valve amplifier and the maintenance of its batteries makes an electroscope more suitable for measurements of a routine nature.

§ 16. ACKNOWLEDGEMENTS

This work was carried out in the Commonwealth X-ray and Radium Laboratory as part of the general campaign against cancer which is being conducted by the Commonwealth Department of Health, and in accordance with the policy of that Department in providing this Laboratory for the investigation of the physical problems involved in the accurate application of radiation in the treatment of disease.

The writer wishes to thank the Director-General of Health, Dr J. H. L. Cumpston, C.M.G., for providing the facilities for this work, and the Natural Philosophy

Department of the University of Melbourne for the loan of a potentiometer. Valuable help was obtained throughout the investigation from Dr C. E. Eddy of this Laboratory. Thanks are also due to Mr J. S. Rogers and Prof. T. H. Laby, F.R.S. of the Natural Philosophy Laboratory, and Dr H. C. Webster, formerly of the Radio Research Board, University of Melbourne, for the opportunity of discussing certain aspects of the work. The writer is particularly grateful to Mr E. L. Sayce, Senior Physicist, and his assistants, of the Metrology Section of the Commonwealth Munitions Supply Laboratories, both for devising the method of measurement of the internal diameters of the metal sheaths used, and for carrying out the great majority of the necessary measurements. Acknowledgement is also made to Mr W. N. Christiansen, Mr C. Barnes and Mr G. E. Shera, of this Laboratory, who respectively assisted in the design and construction of the amplifier, and subsidiary equipment.

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THE APPLICATION OF A NEW PHOTOELECTRIC METHOD TO THE DETERMINATION OF THE OPTICAL CONSTANTS OF SOME PURE METALS

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Communicated by Dr W. H. Taylor, 8 *February* 1939. *Read in title* 9 *June* 1939

ABSTRACT. A modification of the Drude method for the measurement of the optical constants of metals is described, in which the eye is replaced by a photoelectric cell and the extinction position is determined objectively by means of a Lindemann electrometer. By the use of a compensating device, the inaccuracy normally resulting from unsteadiness of the light source is avoided. The results of measurements on copper, nickel, chromium, palladium, and zinc are quoted.

§ 1. INTRODUCTION

When light initially plane-polarized at an azimuthal angle of 45° is reflected from a polished metallic surface, the analysis of the elliptically polarized reflected light provides the data necessary for the calculation of the optical constants of the metal. This method, devised by Drude[1], has previously been applied in this Laboratory to various alloys for wave-lengths within the visible spectrum[2]. The photoelectric instrument described below has been developed primarily in order to extend into the ultra-violet and infra-red regions of the spectrum the range of wave-lengths for which the optical constants may be determined. At the same time the speed and accuracy of measurement in these regions of the spectrum are increased as compared with those achieved in the photographic and bolometric methods previously used, and the rapidity with which it is possible to complete a set of observations covering the whole wave-length range reduces the risk of error due to decay of the polished surface. The new instrument utilizes the same optical system as did the apparatus previously used for visual observations in this Laboratory, but dispenses with the half-shades and replaces the eye by a photoelectric cell. The details given below amplify the brief account already published[3], and refer to the instrument as set up for observations in the visible and infra-red spectral regions.

§ 2. THE APPARATUS AND THE EXPERIMENTAL METHOD

2·1. *The optical system.* The components of the optical system are shown in figure 1. By means of a double monochromator, light of any required wave-length in the range 4000–12,000 A. is focused on slit S_1 and collimated by lens L_1 of the

goniometer G. After plane polarization at an azimuth of 45° by passage through the Glan-Thomson nicol prism N_1, the light is incident on the plane polished surface MS of the specimen, and the reflected elliptically-polarized light passes through the Soleil-Babinet compensator SBC and analysing nicol prism N_2. The emergent light is focused by lens L_2 on slit S_2 and finally falls on the cathode of the photocell C_1. The phase-difference Δ introduced by the compensator in restoring plane-polarization, and the azimuth ψ of the restored plane-polarized light, are the quantities measured experimentally.

The rectangular slit S_2 has four independently adjustable edges and shields the photocell from light forming non-central images due to multiple reflection at the compensator faces.

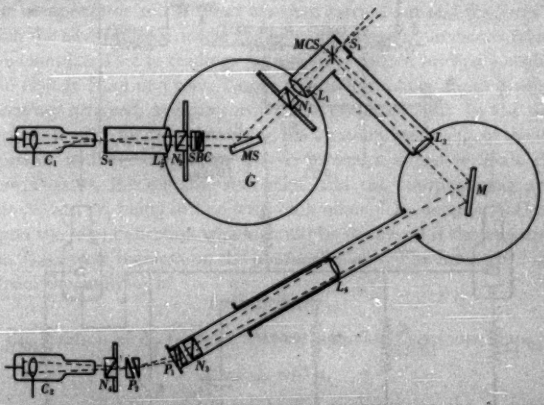

Figure 1. The optical system.

Owing to the impossibility of securing complete extinction, about 1 per cent of background light remains in the central image, and the unsteadiness of all ordinary light-sources prevents an accurate setting for minimum illumination. To remove this difficulty a compensating beam and a second photocell are used. A microscope cover slip MCS placed immediately behind slit S_1 reflects a fraction of the incident light through a collimating lens L_3 to mirror M; this compensating beam then passes in succession through a weak convex lens L_4, a fixed nicol prism N_3, rotary prisms P_1, P_2, and a nicol prism N_4, and finally falls on the cathode of photocell C_2. Lenses L_1, L_2, L_3 are achromatic. The source of light is either a 5-amp. automatic-feed carbon arc or a mercury vapour lamp.

2·2. *The electrical system.* Figure 2 illustrates the photocell and electrometer circuits. The photoelectric currents produce small opposite potentials across the benzene-alcohol high resistance R_1, the contact potential being first balanced out by the subsidiary circuit comprising a 2 v. accumulator of large capacity in series with a fixed resistance R_2 and a variable resistance R_3. The potential across R_1 is recorded

by the needle N of a Lindemann electrometer. If the resistance R_1 is greater than 10^{11} ohms, the time period of the electrometer becomes too large.

Opposite potentials are applied to the plates P_1, P_2 of the electrometer by means of the circuit shown in the upper part of figure 2, in conjunction with a high-tension accumulator of variable voltage. S_1 alters the ratio of the potentials on P_1 and P_2, and so enables the needle to be brought to its mechanical zero. R_4 is the fine zero control. R_5 alters the potentials on the plates without affecting their ratio; it alters

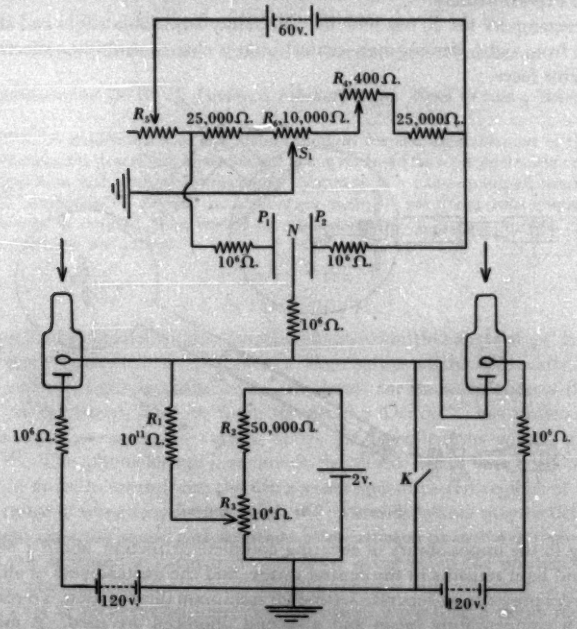

Figure 2. The electrical system.

the sensitivity without affecting the zero position. K is the earthing key. The electrometer needle is observed through a microscope magnifying about 1500 times. The photocells, electrometer, microscope, and resistances are housed in an earthed copper box having several compartments and arranged to rotate with the goniometer telescope arm.

2·3. *Method of observation.* By the use of the telescope, the face of the specimen is first adjusted to lie in a vertical plane parallel with the beam of light and bisecting the field of view. The telescope is then rotated to give the required angle of incidence, and the specimen is adjusted to bring the image of slit S_1 on to the crossed wires.

The telescope is then clamped, and slit S_2 is adjusted to exclude all but the main image.

With a low electrometer-sensitivity, the approximate extinction positions of the compensator and analyser are obtained, the current due to background light being balanced out by adjustment of N_4. With increased electrometer-sensitivity it is necessary to adjust the electrical zero so that it coincides with the mechanical zero, and to balance out the dark current of the photocells and the contact potential of the high resistance R_1. Attention to these details is essential if the electrometer needle is to remain steady at sensitivities of the order of 10,000 divisions per volt. These adjustments are facilitated by the use of a shutter, remotely operated by the observer, which excludes the light from the monochrometer entrance slit. The final adjustments of the compensator and analyser are then carried out and readings are taken, during which the electrometer needle is never more than 20 divisions from the zero position. To complete a set of readings for any wave-length normally requires about 10 min., the change from one wave-length to another taking about 5 min.

2·4. *Accuracy attained, and comparison with visual method.* In the preliminary work, experiments were made to determine the accuracy attainable in the crossing of the nicol prisms with the apparatus shown in figure 1 but without the Soleil-Babinet compensator, the metallic specimen, and the compensating system, the nicol prisms N_1 and N_2 being in line with each other. The accuracy of the settings depends upon the total deflection which would be produced if the maximum intensity of light, I_{max}, were incident on the photocell plate. The intensity I of the light emerging from the analyser is

$$I = I_{max} \sin^2 \alpha,$$

where α is the rotation of N_2 from its true extinguishing position; thus

$$\theta = \theta_{max} \sin^2 \alpha,$$

where θ_{max} is the deflection which would be produced by the maximum light-intensity and θ is the deflection produced by the rotation of N_2 through the angle α from the extinction position. Hence if α_{min} is the angle through which N_2 must be rotated to give an electrometer deflection of one division,

$$\sin \alpha_{min} = \frac{1}{\sqrt{\theta_{max}}}.$$

The accuracy attainable in practice is higher than this, the reason being that a movement of as little as 0·1 of a division is readily observable.

Comparison of two sets of 20 readings of the analyser and compensator, taken for light of wave-length 5780 A. by both methods, a polished nickel surface being used, shows that the mean deviation from the mean of the analyser readings is 9·6′ for the visual method, as compared with 4′ for the photoelectric method; whilst the respective mean deviations of the sets of compensator readings are 0·0036 and 0·0043. Since the constant of the compensator used in the present apparatus is approximately double that of the compensator used in the visual apparatus, the probable error is actually only half that which obtains in the visual method. It is seen that the probable

error in the analyser setting is similarly reduced. More accurate results may therefore be expected from the photoelectric method than were obtainable by the visual method previously used, and this increased accuracy is maintained throughout the whole spectrum—visible, infra-red, and ultra-violet—provided that a suitable photocell is selected for each region.

The Drude theory gives equations for n and k in terms of Δ, ψ, and θ the angle of incidence. From these, partial differential equations can be formed showing the

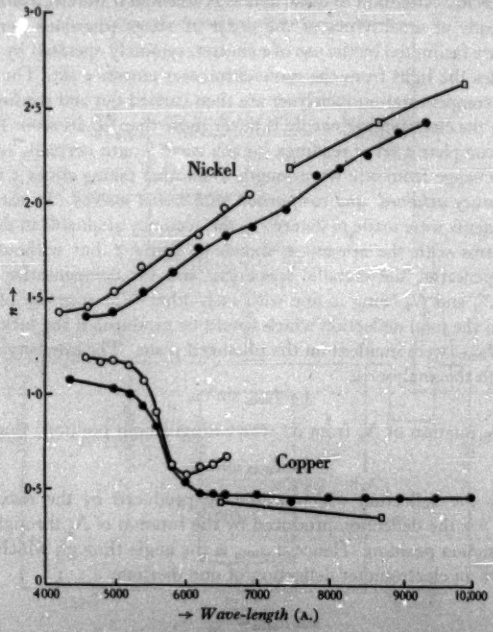

Figure 3. Curves showing variation of refractive index with wave-length for copper and nickel. □ Ingersoll; ○ Tool; ● Authors.

variation of n and k with the three measured quantities. It then follows that errors of 0·0005 in Δ and 5′ in 2ψ when 2ψ is approximately 80° lead to values of dn and dk of 0·01 approximately. Since the assumed errors are larger than often obtain in practice, the apparatus should be capable of yielding results of at least this order of accuracy.

§ 3. MEASUREMENTS ON PURE METALS

The results of measurements of the optical constants of five pure metals—copper, nickel, chromium, palladium and zinc—are included in this paper, and the values obtained for copper and nickel are compared with values previously obtained by

visual methods. Another paper[10] contains the results of measurements on a series of copper-nickel alloys.

3·1. *Copper and nickel.* The specimens were prepared as described by Lowery, Wilkinson and Smare[2] and measurements were made for wave-lengths in the range 4300–9500 A. The variation of the refractive index with wave-length is shown graphically in figure 3, which also includes results previously obtained by other workers[9]. Numerical values are quoted in table 1. The absolute values of the constants are not identical with those given by previous workers, but this is not

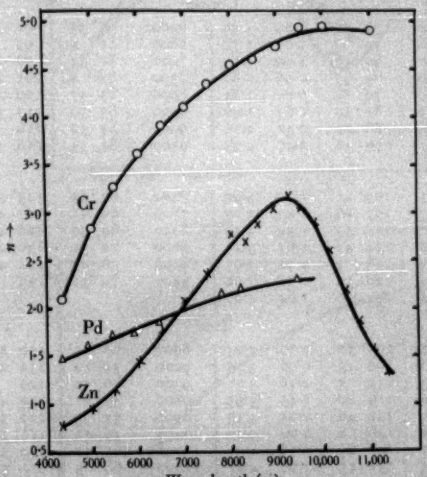

Figure 4.

important, as Lowery, Wilkinson and Smare[2] have shown, provided that the curves of variation with wave-length are parallel, as they are here. The satisfactory agreement between old and new measurements for these two metals confirms that significant results of the expected order of accuracy are obtained with the new photoelectric instrument.

3·2. *Zinc.* A specimen prepared as above was measured over the range 4300–11,300 A., with the results given in table 1, and represented graphically in figures 4–8. It is clear that zinc has an absorption band in the infra-red, nk approaching a maximum at 8750 A.; the curves of n, k, n^2-k^2, show corresponding maxima or minima at 9250, 10,250 and 9800 A. respectively, while the reflection coefficient R has a minimum value at 10,000 A. The only previous measurement, a determination of R by a reflection method by Coblentz[4], also gives a minimum value of R at 10,000 A.

The existence of the infra-red absorption band confirms a suggestion made by

Table 1. 2ψ and Δ represent the experimentally observed settings of the nicol prism and compensator for extinction of the restored plane-polarized light, and n and k the values of refractive index and extinction coefficient calculated from these angles. The angle of incidence θ was 70° in each experiment

Wave-length (A.)	2ψ	Δ	n	k	Wave-length (A.)	2ψ	Δ	n	k
				Copper					
4358	61° 57′	74° 56′	1·07	1·58	6400	80° 35′	92° 33′	0·47	2·50
5000	64 52	79 7	1·03	1·77	6600	81 0	95 0	0·47	2·62
5200	66 7	80 16	1·00	1·83	7000	81 52	99 43	0·46	2·88
5400	67 30	79 33	0·93	1·82	7500	83 14	105 29	0·43	3·22
5600	69 48	79 11	0·84	1·83	8000	83 47	109 23	0·44	3·49
5800	75 45	81 17	0·61	1·96	8500	83 59	113 56	0·45	3·76
6000	78 46	86 26	0·52	2·26	9000	84 32	116 7	0·45	4·00
6200	80 7	89 40	0·47	2·35	9500	84 53	118 42	0·45	4·22
				Nickel					
4600	65° 43′	103° 21′	1·40	2·77	7400	69° 59′	125° 34′	1·99	4·27
5000	67 13	107 26	1·43	3·03	7800	69 51	127 47	2·13	4·43
5400	67 38	111 53	1·54	3·25	8200	69 59	128 25	2·16	4·49
5800	67 50	114 41	1·63	3·44	8600	70 31	130 24	2·24	4·69
6200	68 35	118 1	1·72	3·67	9000	70 23	131 43	2·36	4·79
6600	69 29	120 51	1·78	3·92	9400	70 35	133 5	2·45	4·92
7000	70 1	122 58	1·84	4·07					
				Zinc					
4358	76° 52′	101° 50′	0·77	2·94	8900	62° 25′	132° 47′	3·05	4·15
5000	75 47	109 13	0·96	3·35	9200	59 12	132 23	3·20	3·82
5500	74 39	115 5	1·19	3·71	9500	56 54	129 2	3·05	3·48
6000	73 14	119 40	1·45	3·99	9800	55 11	125 49	2·91	3·23
6500	71 11	122 48	1·75	4·13	10100	55 3	120 32	2·60	3·04
7000	69 18	126 2	2·09	4·23	10400	59 18	116 15	2·19	3·11
7500	67 33	128 29	2·36	4·32	10700	63 52	114 35	1·87	3·18
8000	65 40	132 15	2·79	4·42	11000	68 5	112 54	1·56	3·34
8300	64 31	129 58	2·69	4·18	11300	71 34	114 42	1·39	3·58
8600	63 10	131 27	2·90	4·15					
				Chromium					
4358	68° 13′	125° 20′	2·11	4·15	8000	61° 8′	145° 44′	4·57	4·57
5000	65 50	132 49	2·86	4·44	8500	60 51	146 4	4·61	4·54
5500	64 8	136 33	3·29	4·54	9000	60 22	146 55	4·75	4·46
6000	62 59	139 7	3·64	4·55	9500	60 45	148 37	5·00	4·55
6500	61 49	141 9	3·94	4·50	10000	60 37	148 22	4·95	4·52
7000	61 41	142 31	4·11	4·54	10500	61 27	148 13	4·93	4·68
7500	60 58	144 0	4·36	4·46	11000	60 25	148 33	4·99	4·51
				Palladium					
4358	66° 25′	108° 4′	1·49	3·02	7400	70° 57′	127° 41′	2·03	4·49
4900	67 9	112 51	1·61	3·31	7800	71 41	130 34	2·16	4·79
5461	68 2	117 17	1·73	3·60	8200	72 3	131 58	2·22	4·95
5900	69 16	119 7	1·75	3·82	8600	72 46	132 43	2·20	5·07
6400	69 35	122 41	1·86	4·03	9000	73 27	134 16	2·25	5·29
6900	70 20	125 39	1·96	4·30	9400	74 4	136 39	2·30	5·58

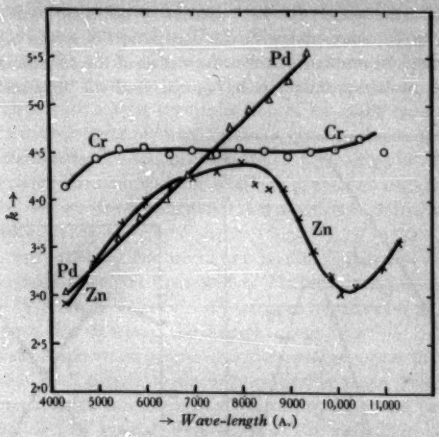

Figure 5.

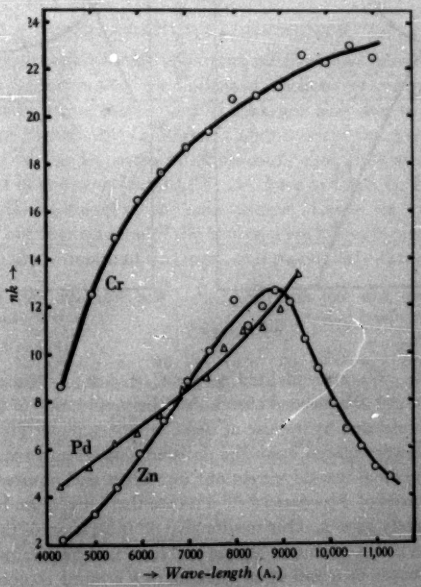

Figure 6.

Mott and Jones[5]; the reason for its existence is not clear, but Mott and Jones suggest that it may be connected with the occupation of zones higher than 4 d.

It should perhaps be emphasized that the values of the constants represent mean values, since the crystal symmetry is hexagonal, and the specimen was large and polycrystalline.

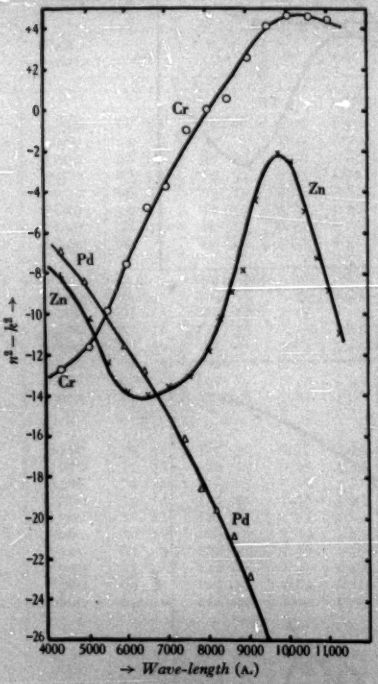

Figure 7.

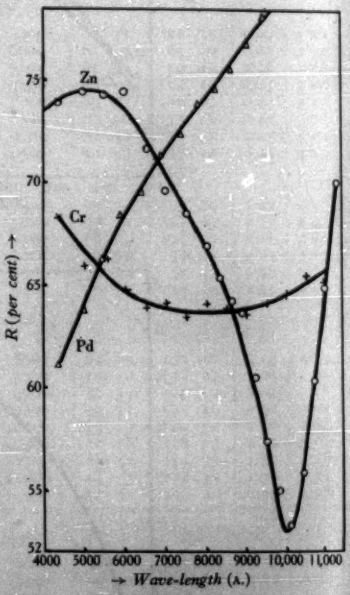

Figure 8.

3·3. *Chromium.* We are indebted to Dr C. Sykes of Messrs Metropolitan-Vickers Ltd. for the gift of a piece of chromium: the preparation of the surface of this hard metal was accelerated by the use of disks of emery paper glued upon circular brass plates which were placed upon the polishing wheel and rotated at a suitable speed. The experimental results are set out in table 1 and figures 4–8.

The measurements of Fréedericksz[6] indicate that chromium has an absorption band at approximately 5500 A. Our results give no indication of the existence of an absorption band between 4000 and 11,000 A. There is, however, a very slight suggestion of a maximum in n and of an increase in k around 10,000 A., while $n^2 - k^2$ has a maximum in the same region: to test the possibility of the existence of an

absorption band in this region would require measurements extending to 13,000 A., outside the range of this instrument. Throughout the greater part of their common range the present results are in close agreement with those of Fréedericksz, but no minimum appears in our curve for nk against λ, which, however, is almost identical with a curve drawn from a trial determination of nk made over the range 4400–9600 A. During the trial determination of the constants, the surface was tested for decay effects by measuring Δ a second time after a lapse of 24 hr. It was found that the difference between the compensator readings in the two instances was less than the probable error, and so it was concluded that a polished chromium surface shows no appreciable decay in 24 hr.

Perhaps the most striking feature of the results is that n^2-k^2 is shown to be positive for wave-lengths greater than 8000 A. This result disproves the assertion of Lowery, Bor and Wilkinson[2] that n^2-k^2 is always negative for metals. Smakula[7] and Fröhlich[8] have also observed the change in sign of n^2-k^2 for silver at 3000 A.

3·4. *Palladium.* A specimen prepared in the usual way gave the results shown in table 1 and figures 4–8. There is no absorption band between 4000 and 9600 A., and if an absorption band exists (as has been suggested by Mott and Jones[5]) it must be sought in regions of still greater wave-length. The palladium surface shows no appreciable decay after exposure for 24 hr.

§ 4. ACKNOWLEDGEMENTS

During the construction of the apparatus we have had valuable help from the staff of the Works Department of the College of Technology and gratefully acknowledge our indebtedness to them. Our thanks are also due to the Manchester University and the Manchester Education Committee for the grant of scholarships and facilities, and to the Department of Scientific and Industrial Research for a maintenance grant to one of us (A. H.). We also have to thank Dr H. Lowery for his assistance in the initial stages of the experiments. Lastly we wish to express to Dr W. H. Taylor our appreciation of the constructive criticism and advice which he has given us generally throughout this work, and particularly during the preparation of this paper.

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THE OPTICAL CONSTANTS OF COPPER-NICKEL ALLOYS

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Communicated by Dr W. H. Taylor, 8 *February* 1939. *Read in title* 9 *June* 1939

ABSTRACT. The paper contains experimental values of the optical constants, for the wave-length range 4000–10,000 A., of copper-nickel alloys ranging in composition from pure copper to pure nickel. The significance of the results is discussed briefly.

§ 1. INTRODUCTION

In the preceding paper, details have been given of a new photoelectric apparatus which has been developed in this Laboratory for the purpose of determining the optical constants of metals over a range of wave-lengths including not only the visible but also the ultra-violet and infra-red regions of the spectrum. In that paper the results of measurements on copper, nickel, zinc, chromium, and palladium were quoted, and it was demonstrated that the new apparatus gives significant results which compare favourably with those obtained by the visual methods used previously.

Measurements have also been made for a series of alloys of copper and nickel over the wave-length range 4000–10,000 A., and it is the purpose of this paper to present the data for these alloys, together with the data for pure copper and pure nickel, in a form suitable for the discussion of the variation of constants with varying wave-length and with varying composition.

§ 2. EXPERIMENTAL METHOD AND MATERIALS

The copper-nickel alloys are particularly suitable for examination of the variation of optical constants with composition, for each of the pure metals crystallizes in a face-centred cubic lattice, the lattice dimensions are nearly the same for both, and the metals form a continuous series of alloys all with the same face-centred structure of phase-sites, with the atoms distributed at random among the phase-sites, and with lattice dimensions which change gradually as the composition is changed. The optical constants deduced from measurements made on a polished surface of a poly-crystalline block are thus characteristic of the alloy, and no complications arise such as might be anticipated in measurements of a non-cubic crystal or a crystal having superlattice structures.

Measurements have been made on pure copper, pure nickel, and thirteen alloys with intermediate compositions, as shown in table 1. Specimens *A* to *F* were

machined from melts prepared *in vacuo* in an induction furnace, and were afterwards polished, annealed to secure homogeneity, repolished, re-annealed to remove cold work, and given a final light polish before measurement; they are rectangular, measuring 3 cm. × 2 cm. × 1 cm. approximately. Specimens *G* to *N* were prepared by Lowery, Bor and Wilkinson for the earlier experiments in which a visual method was used[1], and are considerably smaller, the polished surface being circular with diameters ranging from 1·5 to 2 cm. approximately. Before measurement the surfaces of these specimens were treated with emery to remove the oxide layer formed on prolonged exposure, and polished with magnesia and water: they were not annealed to remove cold work. Both because of the smaller size of the reflecting surface, and because of the simplified treatment of the surface, the results obtained from these alloys may be less accurate than those obtained from specimens *A* to *F*.

Table 1

Specimen	Composition of alloy		Arbitrary constant added in plotting figures 1 to 6		
	Nickel	Copper	*n*	*k*	*nk*
Copper	0	100	−0·4	−1·5	−1·0
A	3·0	97·0	−0·25	−0·5	0
B	5·5	94·5	−0·2	0	0
C	8·5	91·5	−0·1	1·0	0·5
D	11·3	88·7	0	2·0	1·25
E	13·7	86·3	0	2·5	1·0
F	16·5	83·5	0·25	4·0	3·5
G	24·7	75·3	−1·0	−1·7	−1·5
H	35·2	64·8	−1·0	−1·7	−1·0
J	45·1	54·9	−0·5	−1·4	0·5
K	53·2	46·8	−0·4	−1·0	2·0
L	64·9	35·1	0·1	−0·3	5·0
M	76·0	24·0	0·5	0	6·0
N	87·8	12·2	1·0	0·5	9·0
Nickel	100	0	1·5	1·0	12·0

Table 2 contains values of the refractive index *n*, the extinction coefficient *k*, and the absorption coefficient *nk*, for all the alloys for wave-lengths in the range 4000–10,000 A. The variations of *n*, *k* and *nk* with wave-length and with composition are shown in figures 1–7, which also include the data for pure copper and pure nickel quoted in the paper preceding the present one. It should be particularly noted that, in drawing figures 1–6, arbitrary constants have been added in order to separate the curves appearing in each diagram: the values of these constants are quoted in table 1.

§ 3. DISCUSSION OF THE RESULTS

3·1. *The absorption coefficient nk.* Figure 5 shows that the position of the lower wave-length limit of the absorption band of pure copper (at 6000 A.) is practically unaffected by the addition of nickel up to approximately 15 per cent of nickel (between specimen *E* and specimen *F*). Figure 6 shows that for specimens con-

Table 2. 2ψ and Δ (or $180° - \Delta$) represent the experimentally observed settings of the nicol prism and compensator for extinction of the restored plane-polarized light, and n and k the values of refractive index and extinction coefficient calculated from these angles. The angle of incidence (θ) was 70° for specimens A to F and for pure copper and pure nickel, and was 63° for specimens G to N.

Specimens A to F

Composition of alloy	Wave-length (A.)	2ψ	Δ	n	k	nk
Copper 97, nickel 3	4358	64° 36′	72° 52′	0·99	1·56	1·54
	5000	67 21	77 51	0·92	1·76	1·62
	5461	70 51	80 23	0·81	1·88	1·52
	5780	75 41	83 1	0·63	2·03	1·27
	6200	79 5	88 55	0·52	2·31	1·19
	6400	79 33	91 10	0·51	2·42	1·24
	6600	80 15	94 1	0·50	2·56	1·28
	6800	80 21	95 24	0·52	2·63	1·36
	7000	80 58	96 47	0·51	2·71	1·62
	7500	80 56	102 19	0·54	3·02	1·63
	8000	82 11	106 58	0·51	3·31	1·70
	8500	82 9	109 46	0·55	3·50	1·92
	9000	83 44	112 19	0·51	3·70	1·90
	9400	83 44	117 38	0·54	4·10	2·20
Copper 94·5, nickel 5·5	4358	63 50	81 49	1·10	1·86	2·05
	5000	68 19	87 17	0·99	2·13	2·10
	5461	71 6	89 58	0·90	2·28	2·04
	5780	75 48	92 27	0·70	2·45	1·72
	6200	78 52	98 41	0·62	2·79	1·72
	6400	79 16	101 38	0·63	2·96	1·86
	6600	79 41	103 12	0·62	3·05	1·90
	6800	80 33	105 51	0·60	3·22	1·95
	7000	80 31	108 5	0·65	3·42	2·22
	7200	80 48	110 15	0·65	3·51	2·28
	7500	80 25	112 23	0·71	3·66	2·59
	8000	80 59	116 27	0·75	3·97	2·95
	8500	80 53	119 17	0·81	4·20	3·40
	9000	81 38	123 17	0·84	4·58	3·84
	9400	80 53	125 25	0·89	4·78	4·24
Copper 91·5, nickel 8·5	4358	62 43	78 41	1·11	1·74	1·93
	5000	67 16	83 58	0·99	1·98	1·96
	5461	69 18	88 17	0·95	2·18	2·08
	5780	72 17	91 0	0·85	2·34	1·99
	6200	76 3	97 19	0·75	2·69	2·01
	6500	77 4	100 34	0·74	2·87	2·12
	6800	77 50	103 54	0·74	3·07	2·28
	7000	77 54	105 35	0·76	3·16	2·47
	7200	77 43	107 58	0·81	3·32	2·70
	7400	77 34	109 42	0·86	3·42	2·93
	7600	78 7	111 57	0·86	3·58	3·09
	8000	78 14	113 50	0·90	3·71	3·33
	8500	78 44	117 2	0·93	3·97	3·70
	9000	78 41	119 58	1·02	4·20	4·27
	9400	78 14	122 11	1·12	4·38	4·92

Table 2 (*cont.*)

Composition of alloy	Wave-length (A.)	2ψ	Δ	n	k	nk
Copper 88·7, nickel 11·3	4358	61° 44′	75° 19′	1·11	1·60	1·76
	5000	66 12	80 56	1·00	1·86	1·86
	5461	68 42	85 38	0·95	2·06	1·96
	5780	70 23	87 41	0·90	2·17	1·95
	6200	75 15	94 11	0·75	2·52	1·89
	6500	75 42	97 59	0·78	2·72	2·11
	6800	77 1	101 28	0·75	2·92	2·20
	7100	76 48	104 24	0·81	3·08	2·50
	7400	76 47	106 17	0·84	3·19	2·69
	7700	76 56	108 30	0·91	3·41	3·11
	8000	77 3	110 9	0·90	3·44	3·09
	8300	77 19	113 0	0·88	3·64	3·47
	8600	76 50	115 40	1·05	3·81	3·98
	9000	76 49	118 25	1·12	4·02	4·52
	9400	77 48	120 42	1·11	4·23	4·72
Copper 86·3, nickel 13·7	4358	62 55	83 47	1·17	1·92	2·24
	5000	66 29	88 54	1·09	2·17	2·35
	5500	68 50	92 30	1·04	2·35	2·44
	6000	72 38	97 46	0·93	2·66	2·48
	6500	73 48	103 33	0·97	2·98	2·89
	7000	74 18	108 26	1·04	3·27	3·41
	7500	74 6	111 50	1·14	3·48	3·95
	8000	74 41	116 42	1·24	3·82	4·72
	8300	74 26	117 31	1·28	3·87	4·96
	8600	74 25	119 17	1·34	4·00	5·38
	8900	74 49	119 34	1·37	4·08	5·60
	9200	74 24	121 34	1·43	4·18	5·98
	9500	74 29	122 35	1·47	4·27	6·25
Copper 83·5, nickel 16·5	4358	60 2	61 38	1·08	1·18	1·27
	5000	68 42	70 30	0·93	1·48	1·38
	5461	67 38	76 52	0·91	1·72	1·56
	5780	68 46	81 10	0·90	1·89	1·70
	6300	69 35	84 43	0·90	2·04	1·84
	6700	70 0	91 56	0·97	2·35	2·28
	7100	70 30	96 0	1·01	2·54	2·57
	7500	70 31	99 28	1·07	2·70	2·90
	8000	70 52	102 41	1·12	2·87	3·21
	8400	70 31	105 31	1·20	3·01	3·61
	8800	70 8	107 29	1·29	3·10	3·99
	9200	70 41	109 37	1·30	3·24	4·20
	9600	70 31	111 10	1·35	3·33	4·49

taining more than 25 per cent of nickel there is no trace of a minimum in nk in the region of wave-length investigated.

3·2. *The refractive index n.* Figures 1 and 2 suggest that the absorption band is shifted by about 500 A. to slightly shorter wave-lengths as the nickel-content increases from 0 to approximately 17 per cent, and that there is no longer an absorption band in this region for nickel-contents of 25 per cent or more. Figure 7 shows the variation of refractive index with composition for each of the wave-lengths 5000, 6000, 8000 and 10,000 A., and confirms the findings of Lowery, Bor and Wilkinson[3] in experiments covering a more limited range of wave-lengths: the effect of the

Table 2 (*cont.*)

Specimens *G* to *N*

Composition of alloy	Wave-length (A.)	2ψ	$180°-\Delta$	n	k	nk
Copper 75·3, nickel 24·7	4358	64° 24′	78° 45′	1·01	1·71	1·72
	5000	66 55	72 32	1·02	1·96	2·01
	5500	68 43	67 19	1·06	2·20	2·32
	6000	70 8	62 12	1·12	2·46	2·74
	6500	70 39	57 52	1·22	2·68	3·26
	7000	70 44	54 4	1·34	2·88	3·88
	7500	71 29	51 18	1·41	3·07	4·34
	8000	71 13	49 20	1·52	3·18	4·82
	8500	71 59	47 37	1·55	3·32	5·15
	9000	71 50	45 52	1·64	3·41	5·60
	9500	71 57	44 46	1·71	3·51	6·01
	10000	72 40	43 26	1·77	3·64	6·45
Copper 64·8, nickel 35·2	4358	66 12	64 18	1·24	2·25	2·80
	5000	67 43	57 59	1·36	2·57	3·50
	5500	68 14	53 39	1·50	2·80	4·20
	6000	68 27	49 30	1·67	3·02	5·56
	6500	68 32	46 4	1·86	3·22	5·98
	7000	68 40	44 10	1·97	3·33	6·56
	7500	68 40	41 50	2·12	3·46	7·35
	8000	69 26	41 34	2·10	3·54	7·44
	8500	69 22	39 59	2·22	3·64	8·08
	9000	70 17	38 21	2·30	3·83	8·80
	9500	70 28	36 55	2·42	3·94	9·53
	10000	70 52	34 10	2·52	4·43	10·2
Copper 54·9, nickel 45·1	4358	65 45	60 19	1·37	2·40	3·30
	5000	67 9	54 28	1·52	2·71	4·23
	5500	67 28	50 47	1·68	2·89	4·86
	6000	68 5	47 10	1·82	3·13	5·69
	6500	68 23	45 42	1·89	3·23	6·00
	7000	69 3	43 48	1·97	3·38	6·64
	7500	69 3	41 28	2·13	3·52	7·50
	8000	69 30	41 1	2·14	3·58	7·65
	8500	70 30	40 1	2·14	3·73	7·99
	9000	70 59	38 26	2·24	3·88	8·69
	9500	72 9	36 55	2·28	4·11	9·39
	10000	72 32	35 41	2·37	4·25	10·10
Copper 46·8, nickel 53·2	4358	64 39	55 25	1·60	2·56	4·10
	5000	65 47	51 24	1·73	2·79	4·81
	5500	66 8	48 2	1·88	2·96	5·57
	6000	66 53	46 8	1·95	3·11	6·05
	6500	67 15	43 33	2·09	3·26	6·83
	7000	67 41	41 55	2·18	3·53	7·38
	7500	68 47	40 0	2·26	3·59	8·11
	8000	70 54	38 6	2·27	3·90	8·87
	8500	71 48	37 3	2·30	4·06	9·34
	9000	71 49	34 51	2·61	4·24	10·60
	9500	72 12	34 30	2·52	4·31	10·90
	10000	73 29	32 44	2·61	4·62	12·00

Table 2 (*cont.*)

Composition of alloy	Wave-length (A.)	2ψ	$180°-\Delta$	n	k	nk
Copper 35·1, nickel 64·9	4358	64° 11′	55° 35′	1·62	2·53	4·10
	5000	65 6	53 10	1·68	2·68	4·49
	5500	65 55	49 55	1·79	2·86	5·13
	6000	67 4	47 27	1·87	3·06	5·70
	6500	67 51	46 13	1·89	3·16	5·98
	7000	68 11	43 57	2·01	3·31	6·65
	7500	70 9	42 6	2·01	3·57	7·17
	8000	70 40	40 30	2·09	3·71	7·76
	8500	71 33	38 54	2·16	3·90	8·41
	9000	71 54	36 41	2·33	4·10	9·54
	9500	72 30	35 9	2·42	4·29	10·40
	10000	72 40	33 0	2·65	4·45	11·90
Copper 24, nickel 76	4358	65 9	54 34	1·62	2·62	4·23
	5000	66 7	50 39	1·74	2·84	4·91
	5500	67 33	47 46	1·82	3·06	5·57
	6000	68 35	44 38	1·94	3·30	6·40
	6500	69 54	42 36	1·99	3·51	7·01
	7000	70 36	41 4	2·05	3·67	7·53
	7500	71 11	38 54	2·18	3·87	8·45
	8000	71 46	37 7	2·30	4·06	9·31
	8500	72 34	35 23	2·39	4·28	10·20
	9000	73 24	34 4	2·46	4·49	11·10
	9500	74 20	32 31	2·54	4·74	12·10
	10000	75 7	31 1	2·66	5·00	13·30
Copper 12·2, nickel 87·8	4358	66 19	55 54	1·51	2·61	3·94
	5000	68 8	51 48	1·58	2·89	4·57
	5500	69 20	48 7	1·70	3·15	5·34
	6000	70 11	44 51	1·84	3·40	6·24
	6500	71 18	42 1	1·94	3·65	7·07
	7000	72 9	39 52	2·03	3·88	7·88
	7500	72 15	37 51	2·19	4·04	8·84
	8000	73 14	36 36	2·22	4·24	9·41
	8500	73 41	34 49	2·35	4·45	10·50
	9000	74 1	33 27	2·47	4·61	11·40
	9500	74 55	32 17	2·51	4·84	12·10
	10000	75 28	31 49	2·49	4·94	12·30

addition of nickel is very marked for nickel-contents less than about 40 per cent. Mott[2] has suggested that this may be due to the ejection of electrons from the closed shell of nickel.

3·3. *The extinction coefficient k.* Figures 3 and 4 show that the addition of nickel to copper produces a slight though significant change in the position of the minimum in k from 5600 A. in pure copper to 6000 A. in specimen F (approximately 17 per cent of nickel), while no minimum appears from 25 per cent of nickel (specimen G) upwards to pure nickel.

3·4. *Comparison with theory.* Mott[3] has given a formula from which the position of the lower wave-length limit of the absorption band of a copper-nickel alloy may be calculated: for specimen D, for example, with approximately 10 per cent of nickel, the formula predicts a minimum in the nk curve at approximately 8000 A.,

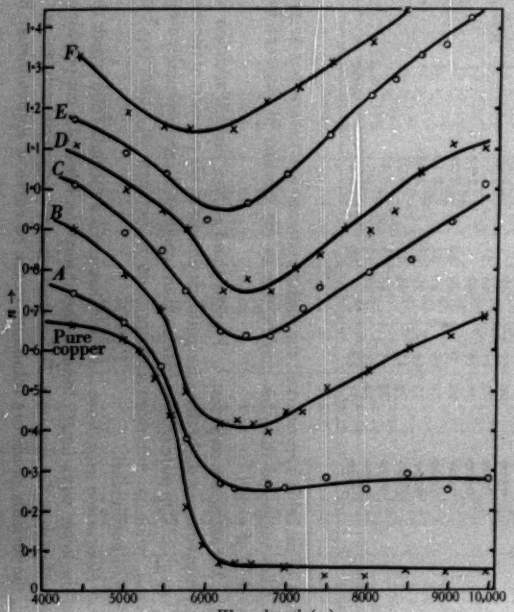

Figure 1.

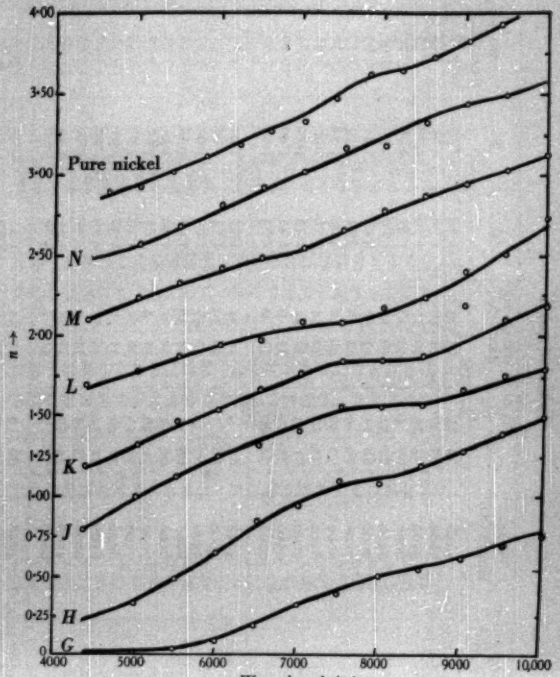

Figure 2.

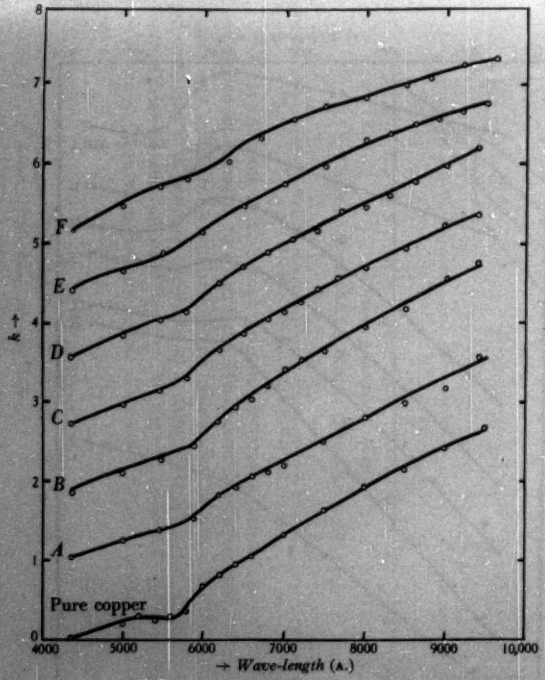

Figure 3.

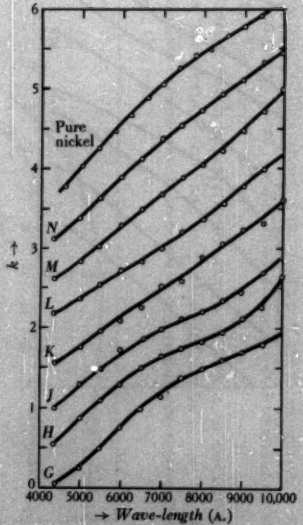

Figure 4.

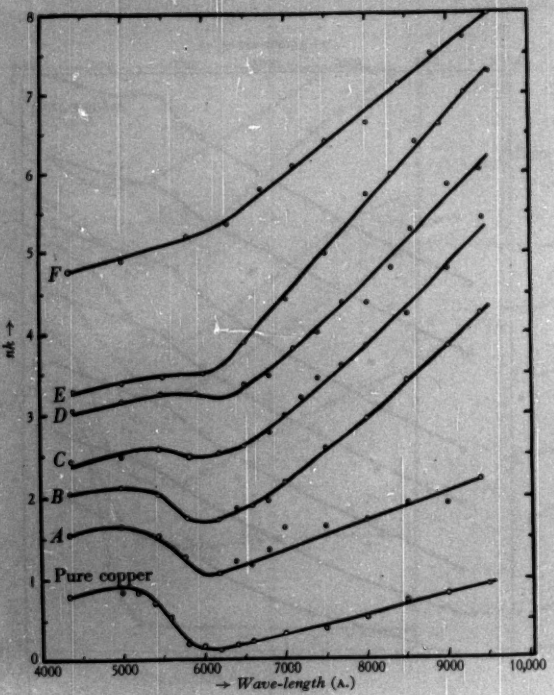

Figure 5.

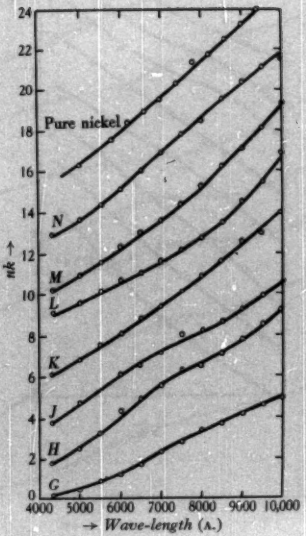

Figure 6.

with further shift of the minimum towards longer wave-lengths as the nickel-content increases.

The observations represented graphically in figure 5 give no indication of a displacement of this order of magnitude: if the position of the minimum shifts at all, it is by not more than 500 A. There is a shift in the minimum of k of about the same order of magnitude.

The slight flattening which appears in the region of 8000 A. in the curves of refractive index for specimens G to N and for pure nickel, figure 2, may be significant, but more accurate experiments than could be attempted with these materials are necessary to test this point.

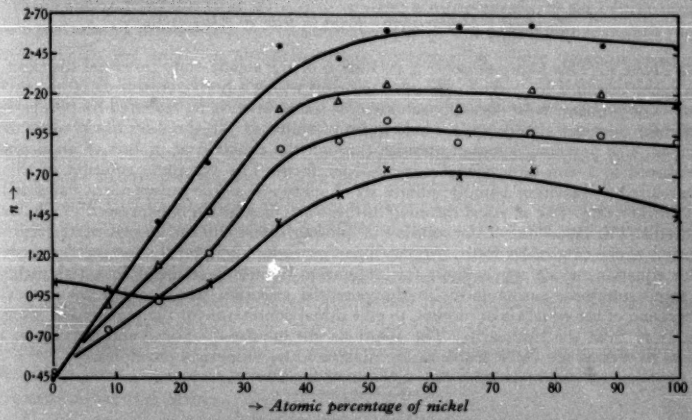

Figure 7. ● 10,000 A.; △ 8000 A.; ○ 6500 A.; × 5000 A.

ACKNOWLEDGEMENTS

We have to thank Dr C. Sykes of Messrs Metropolitan-Vickers Limited for the preparation of alloys A to F, and the Manchester University and Manchester Education Committee for scholarships and facilities for the work. One of us (A. H.) acknowledges with gratitude the award of a maintenance allowance by the Department of Scientific and Industrial Research. We are also indebted to Dr W. H. Taylor for advice and help throughout the course of the work and in the preparation of this paper.

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DISTRIBUTION OF POTENTIAL IN CYLINDRICAL THERMIONIC VALVES

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Received 1 *March* 1939. *Read in title* 26 *May* 1939

ABSTRACT. This paper presents a derivation of the steady-state potential distribution in a space bounded by coaxial cylinders, between which a homogeneous electron current flows. The problem is not formally soluble, but the solution is represented by twenty-four particular solutions covering the whole range of values of the variables likely to occur in practice. The potential equation, derived through the equation of motion of an electron, is reduced to a non-dimensional second-order form. The complete solution of this is obtainable by evaluating a singly infinite set of solutions, as the independent variable has an arbitrary zero. The physical meaning of this form of solution is expressed in similarity theorems. The behaviour of the solution in limiting cases, and the approximate graphical solution of the first-order form, suggested the procedure adopted in solving the second-order equation, which was suitable for integration by means of the differential analyser. In some solutions, a singularity in the potential equation was avoided by numerical integration of the equation of motion, to give initial conditions for the mechanical integration away from the singularity. The solutions are tabulated. Some applications of the results to thermionic valve problems are illustrated by numerical examples.

§ 1. INTRODUCTION

The electric currents in an evacuated space are determined by the geometry and the potentials of the electrodes in the space, and the number of electrons admitted to the space from various electrodes. To determine the relations between the potentials and currents, it is necessary to investigate the motions of the electrons under the influence of a field which itself depends on the distribution of space charge consisting of those same electrons whose motions constitute the currents between the electrodes.

For the one-dimensional case, in which the electrodes are parallel planes and electrons are emitted uniformly from all parts of one plane, the equation of motion of an electron, and, for a steady state, the corresponding equation for the potential distribution, can be solved formally, both for steady-state conditions* and for certain time-varying conditions.†

A cylindrical arrangement of electrodes corresponds much more closely to the

* See, for example, the paper by Fay, Samuel, and Shockley[4] and references therein; and those by Below[1], Fowler, Gossling and Stern[5], Lukoshkow[7].

† See the paper by Benham[2] and references therein.

actual arrangement in many thermionic valves, but the equations for this case are not formally soluble, even for steady-state conditions. Two special solutions have been evaluated numerically by Langmuir and Blodgett[6]; these refer to the particular case of complete space-charge limitation, i.e. the case in which the electric field is zero at the same radius (in practice usually the cathode radius) at which the velocity of the electrons is zero. These solutions are often sufficient for the study of conditions between the cathode of a cylindrical valve and the electrode nearest the cathode, but give no information about conditions outside that second electrode, except in very special cases.

Thus, both in the design of valves and in the analysis of the results of experimental work on them, it would be very useful to have a more extensive set of solutions of the equations of motion (or, for the steady state, of the equivalent equation for the potential) for the axially symmetrical case, covering the whole range of conditions likely to occur in practice. The object of the work described in this paper is to obtain such a set of steady-state solutions. The evaluation of solutions for potentials and currents, varying so rapidly that they change by an appreciable amount during the time taken by an electron to cross the interelectrode space, is a more elaborate and much more extensive problem. Work on it is in progress.

It is true that in an actual valve the equipotentials in the neighbourhood of the grid are not cylindrical surfaces. The effect of this can be made very small,* and in order to determine it from the observed characteristics of a valve, the solution of the equations for the ideal axially symmetrical case is required.

It will be assumed that the electrons emitted from the cathode have all the same velocity. This is not strictly true, but, in many cases, the range of thermal energies is small compared with the energy acquired by an electron in moving between the electrodes, and the use of results referring to a homogeneous electron stream will not involve serious error.

In practice, in the use of valves, the potentials applied to the electrodes are usually the given quantities and the currents are determined, and expressed, as functions of the potentials. But analytically it is inconvenient to solve the equations directly for given values of the potential at different radii; it is much more convenient to take the current, which is independent of the radius, as given, and to determine the potential-distribution. The solution of any practical valve problem is then a matter of interpolation; some examples of this use of the results are given in § 10.

The equation for the potential distribution, in a steady state, is a non-linear second-order differential equation, and can be transformed into a form very convenient, over most of the range, for mechanical integration by means of the differential analyser.† Some of the solutions have a singularity at a point which can conveniently be taken as the starting-point of the integration. Solution in series, or by the use of another form of the equation, is necessary in this neighbourhood, and

* For an examination of this effect in the absence of space charge, see the paper by Rosenhead and Daymond[8].

† See V. Bush[3].

this small part of the calculations was done numerically; otherwise all the solutions here tabulated have been evaluated by the use of the differential analyser at Manchester University.

§ 2. GENERAL THEORY

We consider an axially symmetrical distribution of charge and current, in the absence of a magnetic field, so that the paths of electrons are radially outwards. We first deal with the general case in which the distribution of charge and current is varying with the time, and then obtain the form to which the equations reduce in the steady state. Magnitudes of electrical quantities are supposed to be expressed in electrostatic units.

If, at radius r, the space-charge density is ρ and the velocity of the electrons is v, the convection current i per unit axial length of electrodes is given by

$$i = 2\pi r\rho v. \qquad \text{.....(1)}$$

If E is the field at this radius, the density of the displacement current is $(1/4\pi)\,\partial E/\partial t$, so that J, the sum of the convection and displacement current-densities, is given by

$$J = \rho v + (1/4\pi)\,\partial E/\partial t$$

radially outwards, and I, the sum of the convection and displacement currents per unit axial length, is given by

$$I = 2\pi r J = 2\pi r\,[\rho v + (1/4\pi)\,\partial E/\partial t]. \qquad \text{.....(2)}$$

Since div **J** is identically zero, it follows that

$$\partial I/\partial r = 0, \qquad \text{.....(3)}$$

so that I is a function of t only, and is equal to the current per unit axial length in the bounding electrodes themselves.

Also, the relation between the field and charge distribution is

$$4\pi\rho = \text{div } \mathbf{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE), \qquad \text{.....(4)}$$

since **E** is radial and axially symmetrical; and the relation between the field and the motion of the electron is

$$\frac{d^2r}{dt^2} = \frac{eE}{m}, \qquad \text{.....(5)}$$

where d/dt implies the total rate of change following the motion of an electron.

Multiplying equation (5) by r, and taking the total derivative d/dt, we get

$$\frac{d}{dt}\left(r\frac{d^2r}{dt^2}\right) = \frac{e}{m}\frac{d}{dt}(rE) = \frac{e}{m}\left[\frac{\partial}{\partial t}(rE) + v\frac{\partial}{\partial r}(rE)\right],$$

since v is radial. Substituting from equation (4) we have

$$\frac{d}{dt}\left(r\frac{d^2r}{dt^2}\right) = \frac{e}{m}\left[\frac{\partial}{\partial t}(rE) + 4\pi r\rho v\right]$$

$$= \frac{2e}{m}I(t) \qquad \text{.....(6)}$$

from equation (2). Here I has been written as $I(t)$ to indicate explicitly that it is a function of t only, as has already been shown in equation (3).

When the current in the external circuit, and so I, is varying with the time, the distribution of charge and current in the interelectrode space is a function of the two variables r and t, and it would be expected at first that a partial differential equation would be required to describe it. But equation (6) shows that, by considering the time variation along each electron path, rather than at a point in space, the problem can be reduced to the solution of an ordinary differential equation.

For a steady state, $I=i$ and is constant, so that equation (6) reduces to

$$\frac{d}{dt}\left(r\frac{d^2r}{dt^2}\right)=\frac{2e}{m}i. \qquad \text{.....(7)}$$

Also
$$\frac{d^2r}{dt^2}=\frac{d}{dr}\left(\tfrac{1}{2}v^2\right),$$

and for a steady state
$$\frac{d}{dr}=\frac{\partial}{\partial r} \quad \text{and} \quad \frac{d}{dt}=v\frac{d}{dr}.$$

We will suppose the potential V to be measured from a conventional zero at the cathode, from which the electrons are emitted with zero velocity. Then, for electrons of charge e (which is negative),
$$\tfrac{1}{2}mv^2=|e|V, \qquad \text{.....(8)}$$

and equation (7) can be written alternatively in the form*

$$\frac{d}{dr}\left(r\frac{dV}{dr}\right)=\left(\frac{2m}{|e|}\right)^{\frac{1}{2}}|i|/V^{\frac{1}{2}}. \qquad \text{.....(9)}$$

This equation could be written down more shortly from Poisson's equation and equations (1) and (8); but it is only applicable to a steady state, and we have preferred to derive it from the more general equation (6), through equation (7) which plays an important part in the present work.

§ 3. REDUCTION OF EQUATIONS TO NON-DIMENSIONAL FORM

It is convenient to eliminate the constants e and m, and the current i, by a change of variables which is equivalent to a choice of scale for the measures of r, t, and V. If r_0 is a typical length (for example, the radius of an electrode, or, in the process of solution of the equation, the radius from which the solution is started) we write

$$R=r/r_0, \quad T=\left(\frac{e}{m}\frac{2i}{r_0^2}\right)^{\frac{1}{2}}t, \quad \phi=\left[\frac{9}{4}\left(\frac{2m}{|e|}\right)^{\frac{1}{2}}|i|r_0\right]^{-\frac{4}{3}}V. \qquad \text{.....(10)}$$

These quantities are all non-dimensional.† Then equations (7) and (9) reduce to

$$\frac{d}{dT}\left(R\frac{d^2R}{dT^2}\right)=1, \qquad \text{.....(11)}$$

and
$$\frac{d}{dR}\left(R\frac{d\phi}{dR}\right)=\frac{4}{9}\phi^{-\frac{1}{2}} \qquad \text{.....(12)}$$

* Note that in equation (7), i and e are both negative for an outwards current of electrons.

† i has the dimensions of current ÷ length, since it is the current per unit axial length of the electrodes.

respectively. It is convenient to have a factor $\frac{4}{9}$ on the right-hand side of equation (12) for a reason which will appear later. If we put

$$\xi = \log R, \qquad \text{.....(13)}$$

equation (12) takes the alternative form

$$\frac{d^2\phi}{d\xi^2} = \frac{4}{9}\frac{e^{\xi}}{\phi^{\frac{1}{2}}}. \qquad \text{.....(14)}$$

The general solution of equation (12) involves two arbitrary constants, but one of these is simply a constant of homology, for if $\phi = f(R)$ is a solution of equation (12), then $\phi = a^{-\frac{2}{3}} f(aR)$ is a solution for any constant a, so that the whole field can be covered by evaluating a singly infinite set of solutions for different values of the other constant.

From this form of the solution follow two model, or similarity, theorems relating the steady potentials and currents in different valves, or in the same valve under different conditions, namely: (i)* In any cylindrical valve, if all the currents are multiplied by a factor n, and the potentials at all electrodes by a factor $n^{\frac{2}{3}}$, then the potential at any point in the space is multiplied by the factor $n^{\frac{2}{3}}$. (ii) If a model of any cylindrical valve be made, in which all the radii are multiplied by a factor n, and if the electrode potentials are $n^{\frac{2}{3}}$ times those in the original valve, the currents remaining unchanged, then to potential V at radius r in the first valve there corresponds a potential $n^{\frac{2}{3}}V$ at radius nr in the second valve.

From theorems (i) and (ii) in conjunction it follows that: (iii) in any model of a cylindrical valve, if all the currents are multiplied by a factor n, and all the radii by a factor n^{-1}, the electrode potentials remaining unchanged, then to potential V at radius r in the original valve there corresponds a potential V at radius $n^{-1}r$ in the model.

§ 4. FURTHER REDUCTION OF THE EQUATIONS

From the form $\phi = a^{-\frac{2}{3}} f(aR)$ (with any constant a) for the solutions of equation (12), it follows that the equation for $\phi/R^{\frac{2}{3}}$ is homogeneous in R, and this suggests the use of ξ given by equation (13) as independent variable, and Φ as dependent variable, where

$$\Phi = \phi/R^{\frac{2}{3}} = e^{-2\xi/3}\,\phi. \qquad \text{.....(15)}$$

The use of these variables transforms equation (12) to

$$\frac{d^2\Phi}{d\xi^2} + \frac{4}{3}\frac{d\Phi}{d\xi} + \frac{4}{9}\Phi = \frac{4}{9}\Phi^{-\frac{1}{2}}, \qquad \text{.....(16)}$$

and this form of the equation has been found the most convenient for solution by means of the differential analyser.

We will need the relation between the solution of the reduced form (11) of equation (7), and the solution of the reduced form (12) of equation (9), or the

* It is easy to prove, from the fundamental equations, that this theorem is true for a vacuum thermionic device having any geometry.

equivalent equation (16). From the relations (10), or more directly by making the substitution $T=A\int\phi^{-\frac{1}{2}}dR$, it is found that $A=(2/9)^{\frac{1}{2}}$, so that

$$\left.\begin{aligned}&\phi=(2/9)^{\frac{2}{3}}\,(dR/dT)^{2}, \quad d\phi/dR=2\,(2/9)^{\frac{2}{3}}\,d^{2}R/dT^{2},\\ &\Phi=(2/9)^{\frac{2}{3}}\,R^{-\frac{2}{3}}\,(dR/dT)^{2}, \quad d\Phi/d\xi=2\left[(2/9)^{\frac{2}{3}}\,R^{\frac{1}{3}}\frac{d^{2}R}{dT^{2}}-\tfrac{1}{3}\Phi\right].\end{aligned}\right\} \qquad \text{.....(17)}$$

Further, since ξ does not occur explicitly in equation (16), this equation can be reduced to a first-order equation by the substitution

$$\frac{d\Phi}{d\xi}=p, \quad \frac{d^{2}\Phi}{d\xi^{2}}=p\,\frac{dp}{d\Phi}. \qquad \text{.....(18)}$$

This gives

$$\frac{dp}{d\Phi}=-\frac{4}{3}-\frac{4}{9}\,\frac{\Phi-\Phi^{-\frac{1}{2}}}{p}. \qquad \text{.....(19)}$$

§ 5. BEHAVIOUR OF THE SOLUTION IN SPECIAL CASES

In the evaluation of the solutions, and in the use of the results, it is convenient to know the behaviour of Φ in certain limiting cases, namely (i) in the neighbourhood of a zero of Φ, (ii) when $\Phi\gg 1$, and (iii) when $\xi\to\infty$.

(i) *Near a zero of* Φ. At a zero of Φ (and hence of the electron velocity) the solution of equation (16) has a singularity on account of the $\Phi^{-\frac{1}{2}}$ term, which gives a formal infinity in the space-charge density where the electron velocity is zero. For electrons accelerated outwards from an electrode which they leave at zero velocity, this singularity obviously occurs at a finite value of r; we shall show that it also occurs at a finite value of r, if at all, for electrons retarded as they move outward.

In the neighbourhood of a zero of Φ, the term in $\Phi^{-\frac{1}{2}}/p$ is the leading term on the right-hand side of equation (19).* Omitting the other terms and solving the resulting equation, we have

$$p\sim[\tfrac{16}{9}\Phi^{\frac{1}{2}}+p_{0}^{2}]^{\frac{1}{2}}, \qquad \text{.....(20)}$$

where p_0 is the value of p when $\Phi=0$. Now from the definition of p, it follows that

$$\xi=\int d\Phi/p,$$

and with p given by equation (20), this converges at $\Phi=0$, both for zero and for non-zero values of p_0. Hence the value of ξ at a zero of Φ is finite.

The integration of equation (16) by any method would clearly be troublesome in the immediate neighbourhood of a singularity such as that at $\Phi=0$, especially for small but non-zero values of p_0. It is then convenient to use the reduced form (11) of the equation of motion of an electron, rather than the reduced form (12) of the equation for the potential distribution.

Taking $T=0$ and $R=1$ at the time and radius at which the velocity, and so Φ, are zero, and integrating equation (11) once, we have

$$R\,\frac{d^{2}R}{dT^{2}}=T+\left(\frac{d^{2}R}{dT^{2}}\right)_{0}, \qquad \text{.....(21)}$$

with

$$dR/dT=0, \quad R=1 \quad \text{when} \quad T=0.$$

* It is easily verified that, consistently with this equation, this term cannot be of the order of unity or of any smaller order.

This equation with these initial conditions can be solved in series, or numerically either by direct integration or by iteration. In particular, when $(d^2R/dT^2)_0 = 0$, it becomes

$$\frac{d^2R}{dT^2} = \frac{T}{R},$$

and the first few terms of a solution in series are found to be

$$R = 1 + \tfrac{1}{6}T^3 - \tfrac{1}{180}T^6 + \tfrac{1}{21\,60}T^9 - \tfrac{1}{190\,08}T^{12} + \ldots. \qquad \ldots\ldots(22)$$

(ii) Φ *large*. When $\Phi \gg 1$, the term in $\Phi^{-\frac{1}{2}}$ in equation (16) is small compared with the term in Φ, and may be neglected without serious error; even when Φ is 25 the error is less than 1 per cent. When $\Phi^{-\frac{1}{2}}$ is neglected, the solution of equation (16) becomes

$$\Phi = e^{-2\xi/3}(A + B\xi), \qquad \ldots\ldots(23)$$

where A and B are arbitrary constants, so that

$$\phi = A + B\xi = A' + B \log r. \qquad \ldots\ldots(24)$$

This would be expected, since the $\Phi^{-\frac{1}{2}}$ term in equation (16) is that which arises from the presence of space charge, and, in the absence of space charge, the field of an axially symmetrical distribution of charge has a logarithmic potential.

(iii) ξ *large*. It follows from the results already obtained, that as $\xi \to \infty$, Φ cannot tend to 0 or ∞, for from § 5 (i) it follows that a zero of Φ, if any, must occur at a finite value of ξ; and from § 5 (ii), if $\Phi \gg 1$, it is given by equation (23) and so cannot remain large as $\xi \to \infty$. Hence Φ must tend to a constant value or must oscillate.

If Φ tends to a constant value, this must be 1; this is the reason for the choice of the reduction factor for V in formulae (10) so that a constant $\frac{4}{9}$ appears in equation (12). To find how Φ approaches 1, we put $\Phi = 1 + y$, and neglect y^2. Solving the resulting equation, we find

$$\Phi \sim 1 + Be^{-\frac{1}{3}\xi}\cos\left[(2/9)^{\frac{1}{2}}\xi + \gamma\right]. \qquad \ldots\ldots(25)$$

This shows that Φ oscillates with rather a long period in ξ and a very large damping. The half period is about 6·65 in ξ, corresponding to a ratio of about 1000 to 1 in R, and the decrease in $|\Phi - 1|$ is by a factor of about 85 per half period.

This is closely connected with the oscillation in β^2 found by Langmuir and Blodgett[6] in their solution of equation (9) for electrons starting from rest at the cathode under conditions of complete space-charge limitation. If, for this case, we take r_0 as the cathode radius, we have from equations (10) and (15)

$$V = \left[\frac{9}{4}\left(\frac{2m}{|e|}\right)^{\frac{1}{2}}|i|\,r_0\right]^{\frac{4}{3}}(r/r_0)^{\frac{4}{3}}\Phi,$$

whence

$$|i| = \frac{2}{9}\left(\frac{2|e|}{m}\right)^{\frac{1}{2}}V^{\frac{3}{2}}/r\Phi^{\frac{3}{2}}. \qquad \ldots\ldots(26)$$

But Langmuir defines β^2 by

$$|i| = \frac{2}{9}\left(\frac{2|e|}{m}\right)^{\frac{1}{2}}V^{\frac{3}{2}}/r\beta^2.$$

Hence

$$\beta^2 = \Phi^{\frac{3}{2}}. \qquad \ldots\ldots(27)$$

This oscillation is a curious feature of the solutions of equation (16), as is shown most clearly by consideration of the potential required to produce a given current to an anode, of fixed radius r, in a diode, for different values of the cathode radius r_0 (small compared to r) under conditions of complete space-charge limitation and zero initial velocity. From formula (26) the only effect of the cathode radius r_0 on the $[i, V]$ relation under these conditions arises from the dependence of Φ on (r/r_0), which is oscillatory; hence, as the cathode radius is decreased, the potential required to produce a given current oscillates, instead of varying monotonically as would rather be expected.

§ 6. FIRST-ORDER FORM OF EQUATION

Further light on the general behaviour of the solutions is obtained by considering the first-order equation (19). The general form of the solution of this equation can be sketched by the method of isoclinals. On a diagram of p against Φ, figure 1, are drawn a number of loci on each of which $dp/d\Phi$ is constant, and each of these curves is crossed by a number of short lines at the corresponding value of $dp/d\Phi$. If a curve is then drawn so that, at each point at which it cuts one of the loci, it has the slope of the cross lines on that locus, it is an integral curve of equation (19). Some such curves are sketched in. Near $\Phi=0$ it is difficult to sketch the integral curve from the isoclinals, and the result (20) for the behaviour of p in this region may be used.

The line $p=-\frac{3}{5}\Phi$ is also drawn. Since $\phi^{\frac{1}{2}}$ is proportional to the electron-velocity, it follows that the electron acceleration, and hence the field, is zero where $d\phi/d\xi=0$, and since $\phi=e^{2\xi/3}\,\Phi$, it follows that at such a point $p+\frac{3}{5}\Phi=0$. Hence the point in figure 1 at which an integral curve cuts the line $p=-\frac{3}{5}\Phi$ represents a point in the interelectrode space at which the field is zero. The region of figure 1 lying above this line is the region in which the electrons are accelerated outwards, and the region below it is that in which they are retarded.

Most of the integral curves (all except those which cut the negative p axis) have the general shape of spirals converging rapidly on the singular point $\Phi=1$, $p=0$. Since $\xi=\int d\Phi/p$, and, for clockwise description of the spirals, $d\Phi$ and p have the same sign, it follows that this direction of description corresponds to ξ (and so r) increasing, so the rapid convergence of the spirals to $\Phi=1$ shows that, as $\xi\to\infty$, Φ tends to 1, and oscillates about $\Phi=1$ with rapidly decreasing amplitude, as we have already seen from equation (25).

Of the loci $dp/d\Phi=$ constant, that on which $dp/d\Phi=-\frac{3}{5}$ is of particular interest. Its equation is

$$p=-\tfrac{3}{5}\,(\Phi-\Phi^{-\frac{1}{2}}), \qquad \text{.....(28)}$$

so that it also has the slope $-\frac{3}{5}$ at infinity. Thus there is an integral curve which is asymptotic to this locus (and so also to the locus of zero acceleration $p=-\frac{3}{5}\Phi$) as $\Phi\to\infty$. This curve is marked A in the figure. For this solution $\Phi\sim e^{-2\xi/3}$ when Φ is large, ξ being large and negative, and hence $\phi\to$ constant. In the physical case to which this solution corresponds, the potential near the inner electrode is constant

(since ϕ is constant), and the field there is zero. It follows that the surface charge on the inner electrode must be zero, and the electron velocity must be so high near this electrode that the space-charge density is negligible.

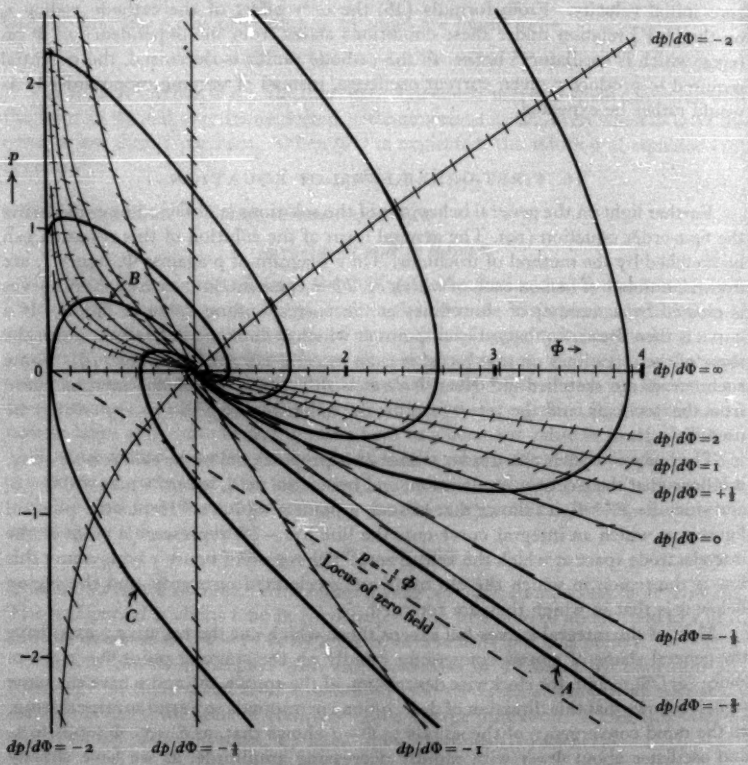

Figure 1. [p, Φ] curves for solution of equation (19).

Further, the locus, equation (28), on which $dp/d\Phi = -\frac{2}{3}$, lies always above the line $p = -\frac{2}{3}\Phi$, and on this line $-dp/d\Phi$ is always greater than $\frac{2}{3}$. It follows that the curve A must always lie above this line, and so cannot reach the axis $\Phi = 0$ in either direction. This is confirmed by actual evaluation of the solution.

Another interesting integral curve is that through the point ($\Phi = 0$, $p = 0$), which has two branches, one (marked B in figure 1) converging to ($\Phi = 1$, $p = 0$) with ξ increasing, and the other (marked C in figure 1) reaching the origin from negative

values of p. Since the point ($\Phi=0$, $p=0$) represents zero velocity and zero acceleration, the two branches of this curve correspond to the two solutions of equation (9) with these conditions evaluated by Langmuir and Blodgett. The first (curve B), in which the electrons are accelerated outwards from a position of zero velocity and zero field, corresponds to the solution distinguished by Langmuir by positive values of β, and applies to the ordinary case of a space-charge-limited diode. The other (curve C), in which the outward-moving electrons are retarded so as to come to rest at a radius at which the field is zero, corresponds to the solution distinguished by Langmuir by negative values of β; we have already shown that this curve cannot coincide with curve A.

Since two integral curves cannot intersect, except perhaps at the singular point, it follows that these three curves A, B, C divide the solutions into groups occupying the regions marked I, II, III in figure 2, in which these limiting curves are the only integral curves shown. Described in the direction of ξ increasing, shown by the

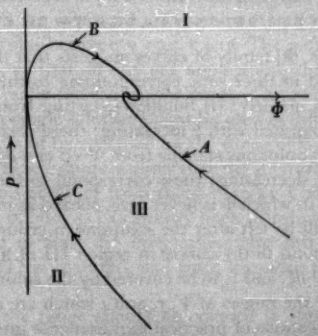

Figure 2.

arrows on curves A, B, C, solutions represented by curves in region I start from zero velocity ($\Phi=0$) at positive values of p and spiral into the singular point ($\Phi=1$, $p=0$); they always lie above the line $p=-\frac{8}{9}\Phi$, so the acceleration is always positive outwards. They apply to the case of the saturated diode with a finite space charge, and to all cases in which a space-charge-limited current has passed a discontinuous increase in field.

Curves in region II also cut the p axis, but at negative values of p, and they lie always below the line $p=-\frac{8}{9}\Phi$, so they represent solutions in which the electrons are always retarded. To obtain such a solution in practice, the electrons must be collected by an outer electrode placed at, or within, the radius at which they would otherwise come to rest, which is always finite by § 5 (i). If there were no such electrode, there would be an inward as well as an outward flow of electrons and no net current.*

* The modification of the argument to make the results applicable to this situation is discussed towards the end of § 10.

Curves in region III do not cut the p axis, and so describe solutions in which the electrons never come to rest; but each curve cuts the line $p = -\frac{1}{3}\Phi$, whence the field is zero at some point on each solution represented by a curve in this region. Such solutions will be required for cases in which a space-charge-limited current has passed through a discontinuous decrease in field; for example, to cases in which grid and anode are at the same potential, since the field must necessarily be zero at some point between them.

Another property of the solutions, which is clear from the form of the curves in figure 1, is that Φ never becomes small (of the order of 1 or less) between two values large compared with 1; hence if $\Phi \gg 1$ at each of two electrodes, then necessarily $\Phi \gg 1$ throughout the space between them, and, by § 5 (ii), the logarithmic formula for the potential can be applied to the space between them.

§ 7. PROCEDURE, AND RANGE TO BE COVERED

The sketch of the $[p, \Phi]$ family of curves suggests that the solutions of equation (16) should be obtained in three sets, corresponding to the three groups of $[p, \Phi]$ curves described in § 6, namely: (i) Solutions starting from $\Phi = 0$ and $d\Phi/d\xi$ zero or positive, at $\xi = 0$, integrated with ξ increasing; these correspond to the curves in region I of figure 2. (ii) Solutions starting from $\Phi = 0$ and $d\Phi/d\xi$ zero or negative, at $\xi = 0$, integrated with ξ decreasing; these correspond to the curves in region II of figure 2. (iii) Solutions in which Φ is never zero, started as may be found convenient to fill the gap which will be left after the solutions in groups (i) and (ii) have been obtained; these correspond to the curves in region III of figure 2.

The ranges of Φ, $d\Phi/d\xi$, and ξ, to be covered by the solutions evaluated, must be decided by considering the ranges of V, r, and i which are of practical interest. In order to convert specifications of practical requirements into limits on the range of the reduced variables Φ etc., we need the relations between these reduced variables and the values of the electrical quantities in practical units.

From formulae (10) we have

$$V \text{ (e.s.u.)} = [(9/4)\,(2m/|\,e\,|)^{\frac{1}{2}}\, r_0 \,|\, i \,|]^{\frac{2}{3}}\, \Phi,$$

e and i being measured in electrostatic units. Now

$$V \text{ (e.s.u.)} = V \text{ (volts)}/300,$$

$$i \text{ (e.s.u.)} = 3 \cdot 10^9\, i \text{ (amp./cm.)},$$

and

$$|\,e\,|/m = 5{\cdot}28 \cdot 10^{17} \text{ e.s.u./g.}$$

Hence when r is measured in centimetres and i in amperes per cm.

$$V \text{ (volts)} = 1667\,(r_0 \,|\, i \,|)^{\frac{2}{3}}\, \phi. \qquad \text{.....(29)}$$

From equation (15), this can also be written

$$V \text{ (volts)} = 1667\,(r \,|\, i \,|)^{\frac{2}{3}}\, \Phi. \qquad \text{.....(30)}$$

The derivative of formula (29) with respect to r, together with formulae (17), gives

the field E in the alternative forms

$$\left.\begin{aligned} E\ (\text{volt/cm.}) &= 1667\ (i^2/r)^{\frac{1}{4}} \left[\frac{d\Phi}{d\xi} + \tfrac{3}{4}\Phi\right] \\ &= 1220\ (i^2/r_0)^{\frac{1}{4}}\ d^2R/dT^2. \end{aligned}\right\} \qquad \text{.....(31)}$$

These relations can alternatively be derived from equation (5), and formulae (10) and (17).

The following typical examples were taken in order to obtain an estimate of the practical range over which solutions are likely to be required: (*a*) A cathode, whose radius r_0 is 0·005 cm., emits electrons with zero velocity ($\Phi = 0$). The current per cm. length is 0·01 amp. The field outside the cathode may have any value from zero to 10^5 v./cm. The radius of the next electrode may be up to 1 cm., so the range of r/r_0 is from 1 to 200, a range of ξ of about 6.

(*b*) A current of 0·01 amp. per unit length passes through an electrode of radius 0·1 cm. This electrode may be at any potential from 0 to 300 v. relative to the cathode, from which the electrons are supposed to be emitted with zero velocity. The field outside it may have any value between $\pm$ 1500 v./cm. The radius of the next electrode is not more than 1 cm., so that, in any one application, the range of r/r_0 is about 1 to 10. But this is not really relevant to the range of ξ to be covered in a solution, as different portions of a single solution might be used in different applications, and the range to be covered is to be specified by the range of Φ rather than of ξ.

With formula (30), these examples show that the interesting range of Φ is from zero up to about 200, and of $d\Phi/d\xi$ from about -50 to $+200$. In example (*a*), the solutions start from $\Phi = 0$, and, as has already been mentioned in § 5 (i), the $[R, T]$ equation (21) was used to start the integrations; for this purpose the range of starting values of d^2R/dT^2 is required, and from formulae (31) this is found to be from 0 to about 300.

It is desirable that the range should be covered in some uniform manner, to facilitate interpolation. For the solutions in sets (i) and (ii) above, started from $\Phi = 0$, this is easily attained by taking a set of starting values of $(d^2R/dT^2)_0$ arranged in a regular way. For group (i), the initial values of d^2R/dT^2 were taken at the values $256/2^n$ ($n = 0, 1, 2, \ldots$) down to 1, and then at 0. It was difficult to decide in advance at what stage to stop the decreasing geometrical progression of the starting values, but it was found that the choice of $(d^2R/dT^2)_0 = 1$, as the smallest non-zero value, was quite suitable. For the set (ii), the starting values of d^2R/dT^2 were taken correspondingly at the values $-32/2^n$ ($n = 0, 1, 2, \ldots$) down to -1, and then 0.

It is not so easy to find an entirely satisfactory way of starting the solutions for region III of figure 2 for which the velocity never becomes zero. At first sight it might seem best to start them at equidistant points on the Φ axis, and to integrate each solution in both directions from its starting-point; but this was found not wholly satisfactory, as solutions started from equidistant points on the Φ axis do not remain nearly equidistant in other parts of the $[\Phi, p]$ field, where they are perhaps more likely to be wanted. It was finally decided to evaluate a set of solutions starting at $\Phi = 60$, with equidistant values of $d\Phi/d\xi$ from -40 to -60.

§ 8. PROCESS OF SOLUTION

The second-order equation (16) is more convenient than the first-order equation (19) for solution by the differential analyser, firstly on account of the difficulty introduced in the solution of equation (19) by the zeros of p, and secondly because the form of result finally required is Φ as a function of ξ, which is given directly by the solution of equation (16), but requires a further integration when the $[\Phi, p]$ relation is found by solution of equation (19).

Since the values of Φ, both in the course of a single solution and in different solutions, may cover a wide range, it is convenient, for solution by the differential analyser, to write equation (16) in the form

$$\left(\frac{d^2}{d\xi^2}+\frac{4}{3}\frac{d}{d\xi}+\frac{4}{9}\right)\frac{\Phi}{\Phi_0}=\frac{4}{9\Phi_0^{\frac{1}{2}}}\left(\frac{\Phi}{\Phi_0}\right)^{-\frac{1}{2}}. \qquad \text{.....(32)}$$

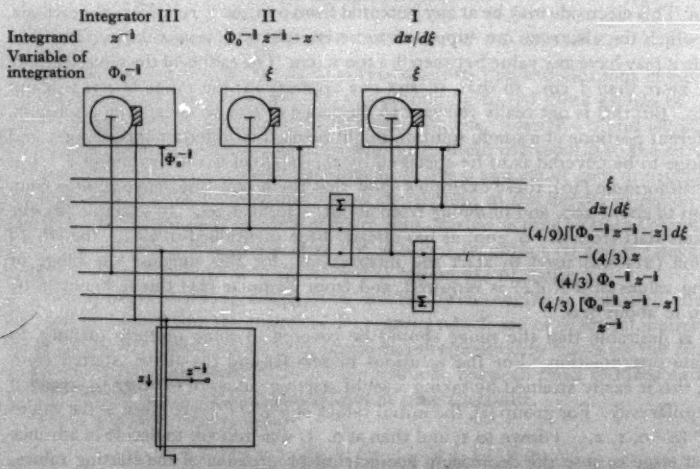

Figure 3. Schematic diagram of set-up of differential analyser for solution of equation (32). ($z=\Phi/\Phi_0$.)

The constant Φ_0 is chosen so that (Φ/Φ_0) covers a convenient range of numerical values in the range over which the solution is being carried out, and so that $\Phi_0^{\frac{1}{2}}$ has a simple numerical value. It may be convenient to change from one value of Φ_0 to another once or twice in the course of the integration, but not more often, since, as we have seen in § 6, once Φ has become small after being large, it cannot become large again.

The differential analyser set-up for equation (32) is shown in schematic form in figure 3, in which Bush's notation for the various units of the machine is used. The solution was recorded on a counter which was read at equal intervals of ξ.

As has already been explained, for solutions starting from $\Phi=0$, equation (21) is used to get away from the singularity before starting the machine solution. A series solution is possible, but, except for the case of complete space-charge limitation, in which $(d^2R/dT^2)_0=0$,* numerical integration is preferable. An iterative process is very rapid and convenient, and converges rapidly over the range required, especially for the higher values of $(d^2R/dT^2)_0$. The numerical solution of equation (21) is taken out until Φ, calculated from dR/dT by formulae (17), becomes at least equal to the smallest value at which it is practicable to start the machine solution, say $\Phi=0{\cdot}05$; the values of Φ and $d\Phi/d\xi$ are calculated from formulae (17), and the machine solution is started from these values. The value of ξ, at which the solution is started, will probably not be a tabular value, and the counter which records the revolutions of the ξ shaft must be set accordingly. Alternatively, the values of dR/dT and d^2R/dT^2 may be calculated at a tabular value of ξ by interpolation, or, better, by a subsidiary integration, so that the machine integration is started at a tabular value. If the same starting value of ξ is used for a series of solutions, a useful cross check on the starting values of Φ and $d\Phi/d\xi$ is provided. The advantages of this procedure were not realized till the work had been in progress for some time, but it was used in the later stages.

For the solutions with $(d^2R/dT^2)_0$ negative, the integration has to be taken for T, and also ξ, decreasing from 0, since the integration is taken backwards from the time and radius at which the electrons would come to rest.

For the solutions corresponding to the curves in region III of figure 2 there is no difficulty at the beginning of the integration, as it is started from definite finite and non-zero values of Φ and $d\Phi/d\xi$.

§ 9. RESULTS

The results are given in tables 1, 2 and 3, for the solutions corresponding to the curves in regions I, II, III of figure 2 respectively. The solutions are numbered to facilitate reference.

In most solutions in tables 1 and 2, Φ reached values 10 to 100 times as great as those at which the machine was started. The effects of small errors, in setting the initial conditions or in integration and particularly in following the curve of $(\Phi/\Phi_0)^{-\frac{1}{2}}$, which was very steep at the beginning of each of these solutions, will build up in much the same way. The result is that, although the variation of Φ with ξ when ξ is well away from zero should always be a solution of the equation to the accuracy of the tabulated values, or nearly so, it may not always be the solution corresponding exactly to the nominal initial conditions. A list of values of Φ, taken across the table at a particular value of ξ, will show small irregularities due to this effect.† These

* See formula (22).

† This cross check was useful in detecting gross errors in a few solutions, usually arising from errors in evaluating the initial conditions for the machine solution from the solution of the $[T, R]$ equation; such solutions were, of course, repeated. In the course of a search for the origin of the errors shown by this check, portions of several of the solutions have been checked by using the tabular values of Φ to calculate ϕ and hence $d^2\phi/d\xi^2$ from equation (14) and thence the second differences of ϕ. The comparison of these with the results of differencing the values of ϕ twice have always been satisfactory.

Table 1. Solutions of equation (16), starting at $\Phi = 0$, with ξ increasing

Solution no.	1	2	3	4	5	6	7	8	9	10
Solution started at d^2R/dT^2	0	1	2	4	8	16	32	64	128	256
Solution started at Φ	0	0	0	0	0	0	0	0	0	0
Solution started at $d\Phi/d\xi$	0	0·734	1·468	2·935	5·87	11·74	23·48	47·0	93·9	187·9
8ξ	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ	Φ
0	0·	0·	0·	0·	0·	0·	0·	0·	0·	0·
0·5	—	0·054	0·094	0·184	0·36	0·72	1·42	2·81	$5{\cdot}6_5$	11·3
1·0	0·0585	0·111	0·188	0·356	0·69	1·38	2·73	5·40	10·8	21·7
1·5	—	0·171	0·277	0·514	1·00	1·98	3·93	7·78	15·6	31·3
2	0·1380	0·228	0·363	0·662	1·28	2·53	$5{\cdot}0_2$	9·95	19·9	40·0
3	0·222	0·338	0·521	0·926	1·78	3·49	6·92	13·70	27·5	55·0
4	0·305	0·437	0·659	1·150	2·19	4·28	8·50	16·86	33·7	67·7
5	0·385	0·532	0·780	1·337	2·53	4·93	9·78	19·39	$38{\cdot}7_5$	77·7
6	0·461	0·616	0·885	1·494	2·81	5·45	10·79	21·40	42·8	85·8
7	0·532	0·692	0·975	1·621	3·03	5·86	11·59	22·98	45·9	92·2
8	0·597	0·759	1·051	1·723	3·20	6·17	12·20	24·19	48·3	96·9
9	0·656	0·818	1·115	1·804	3·32	6·40	12·63	25·02	50·0	100·3
10	0·710	0·870	1·168	1·865	3·41	6·55	12·92	25·59	51·1	102·6
11	0·759	0·915	1·211	1·909	3·47	6·64	13·08	25·90	51·7	103·8
12	0·803	0·955	1·246	1·939	3·50	6·67	13·14	26·00	51·9	104·2
13	0·842	0·989	1·274	1·957	$3{\cdot}50_8$	6·66	13·11	25·92	51·7	103·9
14	0·877	1·017	1·294	1·964	3·49	$6{\cdot}61_8$	13·00	25·69	$51{\cdot}2_5$	102·9
15	0·908	1·041	1·308	1·962	3·46	6·53	12·82	25·33	$50{\cdot}5_6$	101·5
16	0·934	1·061	1·318	1·952	3·41	$6{\cdot}42_8$	12·60	24·87	49·6	99·6
17	0·958	1·077	1·323	1·936	3·36	$6{\cdot}29_8$	12·32	24·31	48·5	$97{\cdot}4_8$
18	0·978	1·090	1·325	1·914	3·29	6·15	12·01	23·69	$47{\cdot}2_5$	$94{\cdot}9_8$
19	0·995	1·100	1·324	1·888	3·22	5·98	11·68	23·01	45·9	92·2
20	1·010	1·107	1·320	1·860	3·14	5·81	11·33	22·30	$44{\cdot}4_0$	$89{\cdot}2_9$
21	1·022	1·113	1·313	1·827	3·05	5·63	10·96	21·55	$42{\cdot}9_8$	$86{\cdot}3_8$
22	1·032	1·116	1·305	1·793	2·96	5·45	10·58	20·78	41·4	83·4
23	1·041	1·118	1·295	1·757	2·87	5·26	10·19	19·99	39·8	80·2
24	1·047	1·119	1·284	1·721	2·78	5·06	9·79	19·20	38·2	77·0
25	1·053	1·118	1·272	1·684	2·69	4·87	9·40	18·41	$36{\cdot}6_9$	$73{\cdot}8_5$
26	1·057	1·115	1·259	1·646	2·59	4·68	9·01	17·62	$33{\cdot}0_5$	70·7
27	1·059	—	1·246	1·609	2·50	4·49	8·62	16·84	33·5	67·6
28	1·061	—	1·233	1·572	2·41	4·30	8·24	16·06	$31{\cdot}9_8$	64·5
29	1·062	—	1·220	1·535	2·32	4·12	7·86	15·31	30·4	61·5
30	1·062	—	1·206	1·499	2·23	3·94	7·49	14·57	$28{\cdot}9_8$	58·6
31	1·062	—	1·193	1·464	2·15	3·77	7·14	13·86	27·5	55·8
32	1·061	—	1·180	1·431	2·07	3·60	6·80	13·17	$26{\cdot}1_8$	53·0
34	—	—	—	1·367	1·91	3·29	6·15	11·86	23·5	47·7
36	—	—	—	1·310	1·77	3·00	5·55	10·64	21·1	42·9
38	—	—	—	1·258	1·65	2·73	5·00	9·53	$18{\cdot}8_8$	38·4
40	—	—	—	1·212	1·54	2·49	4·50	8·52	16·8	34·3
42	—	—	—	1·171	1·44	2·27	4·04	7·60	$14{\cdot}9_8$	30·6
44	—	—	—	$1{\cdot}135_8$	1·35	2·08	3·63	6·77	13·3	27·2
46	—	—	—	$1{\cdot}10_5$	$1{\cdot}27_5$	1·91	$3{\cdot}20_8$	6·02	11·8	24·2
48	—	—	—	1·079	1·21	1·76	2·94	5·36	10·4	21·5
52	—	—	—	1·039	1·10	1·51	2·39	4·24	$8{\cdot}1_5$	16·9
56	—	—	—	1·012	—	$1{\cdot}32_8$	$1{\cdot}97_8$	3·36	6·3	13·3
60	—	—	—	0·993	—	1·19	1·66	2·68	$4{\cdot}9_8$	10·4
64	—	—	—	—	—	1·10	1·42	2·16	$3{\cdot}8_5$	8·2
68	—	—	—	—	—	—	—	1·77	$3{\cdot}0_2$	6·5
72	—	—	—	—	—	—	—	1·48	$2{\cdot}3_8$	$5{\cdot}1_8$
76	—	—	—	—	—	—	—	1·27	$1{\cdot}9_1$	4·1
80	—	—	—	—	—	—	—	1·13	$1{\cdot}5_8$	3·3
84	—	—	—	—	—	—	—	1·05	$1{\cdot}3_0$	2·7
88	—	—	—	—	—	—	—	1·01	$1{\cdot}1_4$	2·3
92	—	—	—	—	—	—	—	0·99	$1{\cdot}0_5$	2·0
96	—	—	—	—	—	—	—	—	$1{\cdot}0_1$	1·7
100	—	—	—	—	—	—	—	—	—	1·5
104	—	—	—	—	—	—	—	—	—	1·4

would be reduced if the integration of the $[T, R]$ equation (12) were carried further out before changing to the $[\xi, \phi]$ equation (16), but in view of the way the results would be used in most applications it did not seem necessary to repeat the mechanical integrations with these better starting conditions.

Table 2. Solutions of equation (16), starting at $\Phi = 0$, with ξ decreasing

Solution no. ...	11	12	13	14	15	16	17
Solution started at d^2R/dT^2	0	−1	−2	−4	−8	−16	−32
Solution started at Φ	0	0	0	0	0	0	0
Solution started at $d\Phi/d\xi$	0	−0·734	−1·468	−2·935	−5·87	−11·74	−23·48
-8ξ	Φ	Φ	Φ	Φ	Φ	Φ	Φ
0	0·	0·	0·	0·	0·	0·	0·
1	0·07	0·13	0·22	0·415	0·80	1·61	3·16
2	0·18	0·30$_5$	0·49$_5$	0·90	1·75	3·49	6·91
3	0·33	0·52	0·83	1·48	2·88	5·67	11·27
4	0·51	0·78	1·22$_8$	2·16	4·21	8·19	16·3$_8$
5	0·73	1·10	1·69	2·95	5·79	11·12	22·2$_5$
6	1·00	1·48	2·23	3·87	7·60	14·5$_8$	29·1
7	1·32$_5$	1·91	2·87	4·92	9·69	18·4	36·9$_8$
8	1·70	2·42	3·60	6·13	12·1	22·9	46·0
9	2·13	3·00	4·44	7·52	14·8$_8$	28·0	56·1
10	2·63	3·67	5·40	9·10	18·0	33·8	67·9
11	3·20	4·44	6·50	10·91	21·7	40·0	81·2
12	3·86	5·32	7·75	12·9$_7$	25·8	47·9	96·3
13	4·61	6·32	9·16	15·3	30·6	56·4	113·0
14	5·47	7·46	10·76	17·9$_8$	36·0	66·0	—
15	6·44	8·75	12·6	20·9$_8$	42·0	76·9	—
16	7·55	10·21	14·6$_5$	24·3	48·9	89·2	—
17	8·80	11·8$_8$	17·0	28·1	—	103·0	—
18	10·22	13·7$_5$	19·6$_5$	32·4	—	—	—
19	11·82	15·8$_8$	22·6	37·2$_5$	—	—	—
20	13·62	18·2	25·9	42·7	—	—	—
21	15·6$_5$	20·8$_5$	29·6	48·7	—	—	—
22	17·9$_8$	23·8$_8$	33·8	55·5$_8$	—	—	—
23	20·5	27·2	38·5	63·2	—	—	—
24	23·4	30·9$_8$	43·7$_5$	—	—	—	—
25	26·6	35·1$_8$	49·6	—	—	—	—
26	30·2	39·8$_5$	56·2	—	—	—	—
27	34·2	45·1	—	—	—	—	—
28	38·7	51·0	—	—	—	—	—

The results have been tabulated with $\xi = 0$ at the initial value of Φ in every case, but it is obvious from the derivation of equation (16), and from the equation itself, that the zero of ξ is arbitrary. To represent the results graphically, in a form convenient for practical use, it is necessary to draw a large number of sets of curves, and this can be done in many ways. The method which seems most convenient is to plot, in each set, all the solutions which pass outwards from a particular value of Φ, this value being different for each set. The sets required would depend on the specific problem to which the results were being applied, so no such graphs of the solutions are given in the present paper, beyond small portions required for illustrating the applications considered in § 10.

Table 3. Solutions of equation (16), starting at $\Phi = 60$, with ξ increasing

Solution no. ...	18	19	20	21	22	23	24
Solution started at Φ / $d\Phi/d\xi$	60 / −40	60 / −45 approx.	60 / −50	60 / −52·5	60 / −55	60 / −57·5	60 / −60
8ξ	Φ	Φ	Φ	Φ	Φ	Φ	Φ
0	60·0	—	60·0	—	—	—	60·0
1	55·2	—	$53·9_8$	—	—	—	52·8
2	50·8	—	48·6	—	—	—	46·5
3	46·7	—	43·8	—	—	—	$40·8_5$
4	42·9	—	39·4	—	—	—	35·8
5	39·5	—	35·4	—	—	—	31·3
6	36·3	—	$31·7_5$	—	—	—	$27·2_5$
7	33·4	—	28·5	—	—	—	23·7
8	30·7	—	25·6	—	—	—	20·5
9	28·3	—	23·0	—	—	—	17·7
10	26·0	—	20·6	—	—	—	15·2
11	24·0	—	18·4	—	—	—	13·0
12	22·1	—	16·5	—	—	—	$11·0_5$
13	20·3	—	$14·7_5$	—	—	—	9·30
14	18·7	—	13·2	—	—	—	7·79
15	17·2	—	$11·7_8$	—	—	—	4·46
16	$15·8_8$	—	$10·5_1$	—	—	—	5·28
17	$14·5_8$	—	$9·3_8$	—	—	—	4·28
18	$13·4_3$	—	8·34	7·09	5·84	4·61	3·39
19	$12·3_7$	9·88	7·42	6·21	5·00	3·81	2·63
20	$11·4_0$	8·99	6·61	5·44	4·26	3·12	1·96
21	$10·5_1$	8·19	5·88	4·75	3·62	2·51	1·39
22	$9·6_8$	7·46	5·22	4·14	3·05	1·98	0·91
23	8·92	6·79	4·63	3·60	2·55	1·53	
24	8·23	6·19	4·12	3·13	2·12	1·15	
25	7·59	5·64	3·65	2·71	1·75	0·82	
26	7·00	5·14	3·22	2·34	1·42	0·54	Reaches
27	6·46	4·69	2·86	2·01	1·14	0·32	zero
27·5						0·22	and
28	5·97	4·29	2·53	1·73	0·91	0·13	stops
28·5						0·05	
29	5·51	3·92	2·24	1·49	0·71	Reaches	
30	5·09	3·58	1·98	1·28	0·34	zero	
31	4·71	3·27	1·75	1·09	0·41	and	
32	4·36	3·00	1·56	0·94	0·30	stops	
34	3·74	2·53	1·24	0·70	0·16		
36	3·22	2·15	0·99	0·34	0·10		
38	2·78	1·84	0·82	0·44	0·13		
40	2·42	1·59	0·70	0·39	0·19		
42	2·12	1·39	0·63	0·39	0·29		
44	1·87	1·23	0·59	0·41	0·40		
46	1·66	1·11	0·58	0·45	0·50		
48	1·49	1·03	0·59	0·50	0·60		
50	—	—	—	0·55	0·69		
52	1·24	0·91	0·65	0·61	0·75		
54	—	—	—	0·67	0·81		
56	1·08	0·86	0·72	0·73	0·87		
58	—	—	—	0·78	0·91		
60	0·99	0·85	0·80	0·82	0·94		
62	—	—	—	0·85	0·98		
64	0·94	0·86	0·86	0·88	—		
66	—	—	—	0·91	—		
68	0·92	0·88	0·90	0·94	—		
70	—	—	—	0·95	—		
72	0·92	0·91	0·94	0·97	—		
74	—	—	—	0·98	—		
76	0·92	0·94	$0·95_8$	0·99	—		
78	—	—	—	0·99	—		
80	0·93	$0·96_4$	0·96	—	—		
82	—	—	—	—	—		
84	—	0·98	—	—	—		

§ 10. EXAMPLES OF APPLICATION OF THE RESULTS

We do not propose to give here an exhaustive account of the possible applications of the results, but we will consider two particular cases as examples of their use, namely, the problem of finding the field-strength at the cathode of a saturated diode and the problem of finding the position and depth of the potential-minimum between a cylindrical screen and anode at the same potential; both of these problems may be solved directly from the results of the present work. (i) A diode consists of a straight filament 0·4 mm. in diameter surrounded by a cylindrical anode 2 cm. in diameter. The current per cm. of axial length is 1 mA. and the potential-difference between anode and cathode is 50 v. It is required to find the field-strength at the surface of the filament. From formula (30), at the filament ($\xi = 0$, say), $\Phi = 0$; and at the anode, where $\xi = \log_e (2/0{\cdot}04) = 3{\cdot}91$, $\Phi = 3{\cdot}0$. On the $[\xi, \Phi]$ diagram covering

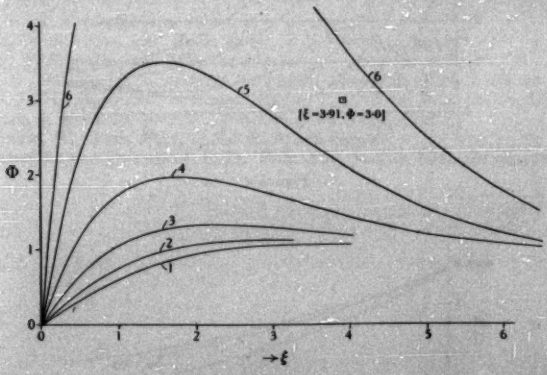

Figure 4.

this range, drawn with all the curves through $\Phi = 0$ at $\xi = 0$, shown in figure 4, the point (3·91, 3) lies between the curves 5 and 6.* Interpolation of the curve which passes through this point gives 9·0 as the value of $d\Phi/d\xi$ at the cathode, whence, from formula (31), the field at the cathode is 55 v./cm.

(ii) The outer electrodes of a cylindrical valve are a screen 0·4 cm. in radius, and an anode 1 cm. in radius. Both electrodes are at a potential of 200 v. relative to the cathode, and the current between them is 27 mA. per cm. length. It is required to find the potential distribution between screen and anode. From formula (30), at the screen ($\xi = 0$, say), $\Phi = \Phi_S = 2{\cdot}46$; and at the anode ($\xi = 0{\cdot}916$), $\Phi = \Phi_A = 1{\cdot}33$. It is seen from figure 5 that the Φ curve which passes through the values Φ_S and Φ_A at points separated by 0·916 in ξ must lie between the solutions 19 and 20; these are therefore redrawn in figure 6, starting together at $\Phi = \Phi_S = 2{\cdot}46$ and $\xi = 0$. The curve passing through the point (0·916, 1·33) has been interpolated, and the value

* The numbering of the curves corresponds to the numbering of the solutions in tables 1, 2 and 3.

of the potential at any radius between the electrodes may then be found from this curve and formula (30).

Another possible application of the results would be the construction of tables giving the relation between T and R for each of the various solutions, equation (17) being used; from such tables, and the definitions of T and R as in formulae (10), steady-state transit times in practical cases could be deduced. Construction of such tables would be rather lengthy and laborious, and has not been carried out; if it had

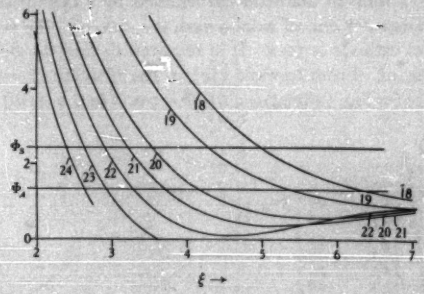

Figure 5.

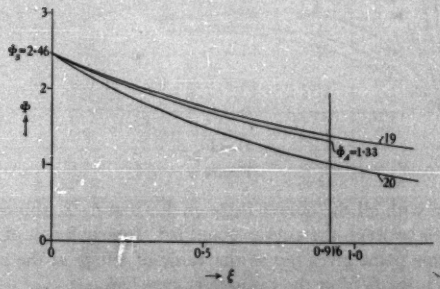

Figure 6.

been contemplated, it would probably have been better to take the integration of the $[T, R]$ equation (11) out from a zero of Φ rather further than was actually done before changing over to the use of equation (12).

Although it has been assumed throughout that the electrons move in the direction of r increasing, the equations, and solutions, hold for electrons moving outwards or inwards or both, provided that all electrons having the same velocity are at points at which the potential is the same. When there are electrons moving in both directions, i must be taken to mean the sum of the magnitudes of the inward and outward currents.

If, at some value of the radius, $\Phi = 0$ and $d\Phi/d\xi = 0$, some electrons may be turned back there and some not. Up to this value of r, solution (11) holds; beyond it solution (1) holds. The application of the results to this case is analogous to that described by Fay, Samuel, and Shockley[4] for the similar problem in the case of plane parallel electrodes.

§ 11. ACKNOWLEDGEMENT

The authors desire to thank The General Electric Company Limited and the Marconiphone Company, in connexion with whose research this work was carried out.

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THE RECEPTION OF BROADCAST SIGNALS SUBJECT TO DISTURBANCE BY ATMOSPHERICS

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Received 9 *June* 1939.

ABSTRACT. In many parts of the world the chief hindrance to radio reception is atmospheric disturbance. The paper describes a machine which produces pulses similar to atmospherics, and records speech-intelligibility tests carried out with it with a view to finding how intelligibility varies with atmospheric noise. It is suggested that a signal/atmospheric ratio equal to 20 db., corresponding to a voltage ratio of 10 : 1, should be a sufficiently high standard for districts where atmospheric disturbance is severe. The atmospheric level has been measured with a special peak voltmeter which integrates the value over several seconds.

§ 1. INTRODUCTION

In considering any scheme for broadcasting in tropical or subtropical countries, the most important hindrance to a satisfactory service is atmospheric disturbance. There seems, however, to be some uncertainty[1] as to the radio signal-strength necessary to produce satisfactory reception for a given value of atmospheric disturbance.

As a result of work on electrical interference carried out under the control of the International Special Committee on Radio Interference (C.I.S.P.R.), a signal/interference ratio equal to 40 db. is considered acceptable[2]. This figure refers to interference with broadcast reception caused by electrical appliances—a form of disturbance considerably different from that produced by atmospherics—and may require modification before it can be used to define acceptable standards of signal-to-atmospheric strength. It has been recognized that Colonial Administrations might well be satisfied with less stringent standards[3], and in any case the standard is impossibly high for reception in most parts of India. With a view, therefore, to defining some lower and more practicable standard for tropical conditions, speech-intelligibility tests have been carried out under conditions of varying noise similar to atmospheric disturbance.

§ 2. PRODUCTION OF ATMOSPHERICS

Atmospheric disturbance being such a variable quantity, and difficult to obtain unadulterated, it was decided to produce similar effects artificially. As subjective testing methods were to be used, the primary test applied was that the artificial

atmospherics should appear to the ear like actual atmospheric disturbance. That is to say, there should be a background of crackling with irregular loud splashes of noise.

The machine shown in figure 1 produces pulses which, when rectified and amplified, have satisfied listeners with their similarity to atmospheric disturbance from a broadcast receiver. Two steel balls *A* and *B* press lightly against a steel disk *D*, 30 cm. in diameter, which rotates once a minute. With the circuit as drawn, oscillographic investigation has shown that damped oscillations are set up sometimes in circuit *E* and sometimes in circuit *F*. Owing to the uneven nature of the contacts between the steel balls and the disk, the pulses are set up irregularly in time, and are of varying amplitude. The mean number of individual pulses

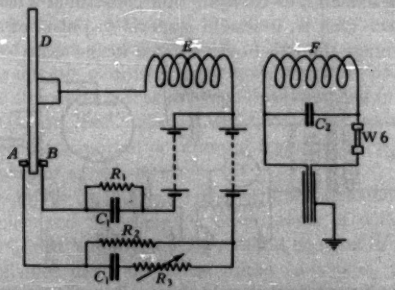

Figure 1. Apparatus for imitating atmospherics. R_1, 5000 Ω.; R_2, 10,000 Ω.; R_3, 2000 Ω.; C_1, 0·25 μF.; C_2, 0·001 μF.; W_6, Westector no. W 6.

with both contacts is about 120 per sec. This compares favourably with Norinder's[4] value of between 1 and 2 for the most frequent number of atmospherics per 10^{-2} sec.

The frequency of 50 per cent of the pulses is that of the circuits *E*–*A* and *E*–*B*, and is of the order of 12 kc./sec. The most common frequencies occurring in periodic atmospherics (the most common form), according to Appleton, Watson Watt and Herd[5], and also to Norinder, are of the order of 10 kc./sec. which shows a fairly close correspondence.

The duration of the damped oscillations in this circuit is about 1000 μsec. Efforts were made, by putting resistances across coil *E*, to reduce this time to 300 μsec., the most common duration for periodic atmospherics, but with detrimental effects on the sound from the loudspeaker.

Circuit F–C_2 represents an attempt to imitate the high-frequency embroidery[6] of atmospherics of the lightning type. The frequency of the pulses produced by it is 65 kc./sec. and the duration 1500 μsec.

In order to produce the effect of continuous background noise with splashes of much larger amplitude, a piece of paper with irregular perforations is attached to the steel disk between contact *B* and the disk. By adjustment of R_3 the effect of the continuous pulses from *A* was made to provide the background while *B*

produced the larger splashes, which occurred about 40 times per minute. The mean duration of contact B through the perforations was 0·7 sec., and the ratio of splash peak voltage to background peak voltage could be varied from 12 : 1 to 1 : 1.

§ 3. THE MEASUREMENT OF NOISE-LEVEL AND SPEECH-LEVEL

In work on telephone articulation[7] a special noise-meter is used which weights the noise-frequencies according to their degree of interference with speech. As, however, this work was primarily concerned with broadcast standards, a meter with a practically constant frequency response from 50 to 10,000 c./sec. was used.

The International Special Committee on Radio Interference has designed a peak voltmeter for measuring electrical interference[8], the principal feature of which is that it should have a small grid-charging time constant (1·0 msec.) and a discharge constant of 160 msec. That is, it should respond to pulses of short duration, and at the same time average the effect over a much longer time.

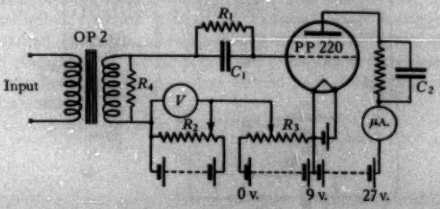

Figure 2. Slide-back voltmeter for peak atmospheric voltage measurements. R_1, 100,000 Ω.; R_2, 25,000 Ω.; R_3, 50,000 Ω.; R_4, 15,000 Ω.; C_1, 0·004 μF.; C_2, 4·0 μF.

In measuring atmospheric disturbance, which may consist of widely separated pulses with large variations in amplitude, some such averaging device is necessary, so that the meter sets itself somewhere between the peak background reading and the peak splash reading. If the discharge constant of the meter is made too great, the needle will rise and remain at the peak value for a splash. If it is made too low, the needle will fluctuate too rapidly to be read. The exact value of this discharge constant must be determined subjectively. If it is too large, the meter gives a reading which is too high and not a measure of the disturbance value of the atmospheric. With a correct discharge constant, varying the ratio of background noise to splash, while the subjective disturbance value is kept constant, will cause no change in the voltmeter reading. It is not necessary for this adjustment to be carried out with a high degree of accuracy.

Figure 2 represents a form of slide-back voltmeter which fulfilled these conditions. The anode current was backed off to an average value of 4·0 μA. on a unipivot instrument in all measurements, the peak voltage being measured on the voltmeter in the grid circuit.

The grid time-constant was 0·43 msec. and the discharge time-constant of the microammeter circuit was 4·0 sec. The effect of a pulse on the meter was to cause a rapid movement upward of the microammeter needle, and a very slow fall back.

That the meter did actually register peak voltages was checked by comparison with a C.R. oscillograph and a commercial peak voltmeter. No calibration of the meter was necessary insomuch as in the method followed the meter was used simply for bringing noise and speech levels to a predetermined value.

§ 4. EXPERIMENTAL PROCEDURE

The pulses from the atmospheric machine were rectified and fed by screened leads to a 40-db. attenuator and then on to the mixer stage of an amplifier. Speech output from a crystal microphone housed in a large padded box was passed through a preamplifier, mixed with the disturbance, and fed into a large loudspeaker. All measurements were made at the output through a step up (25 : 1) transformer. The loudspeaker and amplifier response were tested for normality.

In making measurements, first of all the effect of hum and stray microphone pick-up was balanced out on potentiometer R_3 of the voltmeter; it was assumed that noise interference is directly additive[9] and would therefore only introduce a small constant error. The atmospheric machine was then switched in and the output was brought up to a standard level. Speech was independently brought to the same level and attenuation then inserted or removed from the attenuator to give various noise levels on a constant speech-level.

In order to measure the intelligibility of speech, the articulation, testing methods developed by Fletcher and Steinberg[10], descriptions of which may be found in several papers[11], [12], were employed. The method consists in transmitting a series of meaningless syllables made up of an initial consonant, a vowel and a final consonant, mixing them with the required level of noise, and arranging for listeners to record their impressions. A quantity called the *syllable articulation*, defined as the percentage of the total number of syllables correctly observed, was calculated from the formula VC^2, where V and C are the ratios respectively of vowels and consonants correctly heard.

Curve 1, figure 3, shows the effect of atmospheric noise on syllable articulation. For comparison, curve 2, due to Knudsen[12], shows the effect of noise on the articulation of speech in an auditorium. Curve 3 gives the articulation value for zero atmospheric noise, and may be regarded as the reference value for the system.

Fletcher and Steinberg give in their paper a graph connecting syllable articulation with what they call *discrete sentence intelligibility*. Curve 4 is derived from curve 1 by using this graph, and may be said to represent the effect of atmospheric noise on the intelligibility of ordinary speech. It would not represent the intelligibility of speech with large numbers of unfamiliar words.

Each point on the fundamental curve 1 represents the recording of 330 syllables. Various haphazard factors tend to cause a large spread in the results; it is possible, for instance, for each syllable in one group to be spoken at the same time as a splash, and this group would therefore give a low value of syllable articulation. Varying room-noise, changing ratio between background and splash, and practice effects in the observers, contributed in a less degree. The calculated probable error of a single point is in the neighbourhood of ± 2 per cent.

Opinions were asked with a view to deciding which was the maximum tolerable noise-level, for it was realized that speech may be intelligible and yet intolerably noisy to listen to in comfort. For this test a reasonably interesting passage from a book was read. The general opinion was that, with equal levels of speech and noise, only interesting subject-matter would be listened to. With noise-level 10 db. below speech-level, conditions were easily tolerable, and at 20 db. below speech-level the noise did not inconvenience listeners at all.

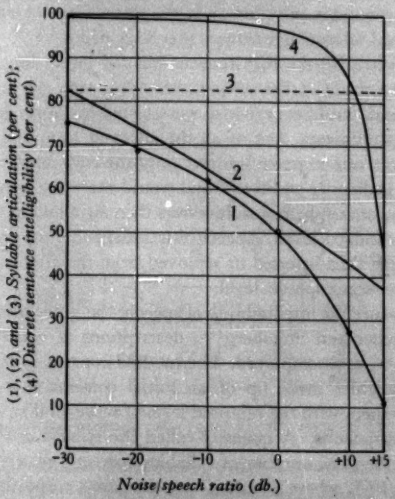

Figure 3. Syllable-articulation and discrete-sentence intelligibility plotted against atmospheric noise-level.

§ 5. DISCUSSION OF RESULTS

Comparison of the present results with those obtained by Knudsen for auditorium noise shows results of the same order. It is interesting to note, however, that high levels of atmospheric disturbance cause a rapid deterioration of speech intelligibility, and also that quite small amounts of noise (−30 db.) cause noticeable loss of syllable articulation.

Curve 3 may be said to represent the overall articulation of the system of speakers, language, apparatus and observers, with zero atmospheric disturbance. Comparison of the overall articulation in another system with this value enables a syllable articulation to be plotted against atmospheric noise. For instance, it might be found that the Sanskrit and Persian consonants in use in India might give a low value of the overall articulation of a broadcast system. Reference to curve 3 would show the amount by which the articulation curve 1 had to be shifted downwards to allow for the effect of the new consonants. Actually a test was made

with syllables made up of Sanskrit and Persian sounds but the test was not completed, although the evidence obtained made it appear that the difference in articulation would not be very great.

From curve 1 it is seen that with a signal/atmospheric noise ratio equal to 20 db. (corresponding to a voltage ratio of 10 : 1) the articulation is 68 per cent. Knudsen estimates that speech articulation of 75 per cent is acceptable, while other writers[11] consider articulation values between 70 and 80 per cent good, and ample for the general intelligibility of speech. The discrete sentence intelligibility for this level is 99 per cent from curve 4, and it is therefore suggested that for broadcasting in tropical and sub-tropical regions an average signal/atmospheric noise ratio equal to 20 db. should be regarded as acceptable. The atmospheric noise must, of course, be integrated by some form of peak voltmeter, such as that described.

§ 6. ACKNOWLEDGEMENTS

The author wishes to thank Prof. Hollingworth and Mr N. C. Stamford for continual advice and interest. He wishes also to thank Messrs B. Craven, D. Quin and T. Yearsley for helping as listeners: and the Church Missionary Society for permitting the author to use his leave for carrying out this work.

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EXCITATION OF THE GREEN AURORAL LINE OF OXYGEN

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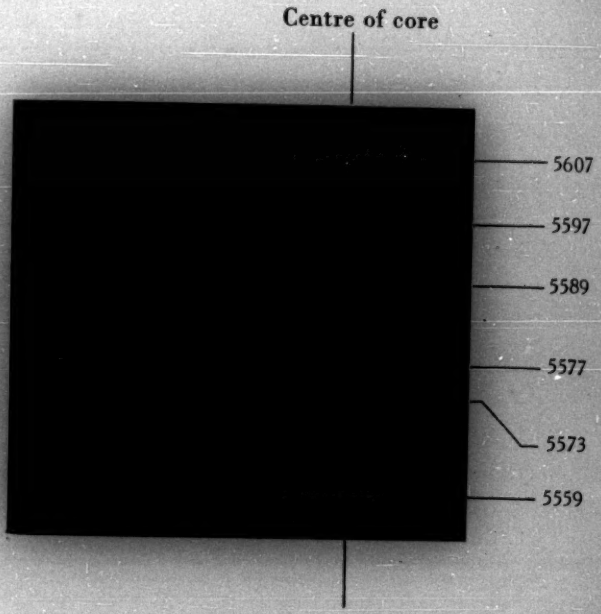
Communicated by Professor W. B. Morton 25 March 1939

ABSTRACT. The excitation of the green auroral line of oxygen has been studied in a number of discharge tubes containing mixtures of argon and oxygen, and the early results obtained by McLennan for the optimum conditions for obtaining the line have been confirmed. It has been shown that, with discharge tubes passing heavy currents, the line is emitted mainly from an aureole surrounding the central, heavily ionized part of the gas, and the bearing of this result on the production of other forbidden lines is pointed out. The collision processes involved in the production of metastable oxygen atoms and in the quenching of the auroral line by certain impurities have been considered, and the probable importance of resonant collisions with metastable argon atoms established.

§1. INTRODUCTION

KNOWLEDGE of the internal electrical state of discharge tubes has now reached a stage at which the essential features of several classes of arcs and glows can be regarded as established. Little is known, however, about conditions in the majority of discharges used for excitation of light in spectroscopy. Investigations have now been made in this laboratory on a number of tubes which emit, or might have been expected to emit, spectral lines produced in transitions forbidden to a dipole approximation. These have led to the recognition, although not yet to the complete solution, of many of the difficulties likely to be encountered in an attempt to obtain a full analysis of the state of any electrically excited spectroscopic source. The present paper contains an account of this work, with particular reference to the excitation of the green auroral line (5577·35 A.) of oxygen in presence of argon. This line is a prominent feature of the spectrum of the upper air. It is emitted in a transition between the low metastable ^1S and ^1D states of the neutral atom. These have excitation potentials of 4·2 and 2·0 V. respectively from the ^3P ground state, all five terms belonging to the $2s^2 2p^4$ electron configuration. The transition probability for production of the green line has been calculated to be 2·0 sec^{-1}, and that for the ultraviolet transauroral line (2972 A., $^3P_1$–^1S) to be 0·2 sec^{-1} [1]. The latter has not been identified with certainty in the spectum of the upper air, although two red lines emitted in transitions between the ^1D and ^3P states are present. All these lines have been produced in the laboratory, the green line first by McLennan and his collaborators [10], the red lines by Hopfield [9] and the



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ultra-violet line by McLennan[14]. Our experiments and conclusions are largely extensions of those of McLennan. From the behaviour of our tubes, together with the theory of positive columns, and the descriptions of the tubes used by McLennan and others, we have been able to form what appears to be a reasonable picture both of the macroscopic electrical processes and of the molecular processes involved in the production of atoms in the ^1S state. The main facts which have emerged are on the electrical side the importance of regions of low ionization, such as the aureole which surrounds the core in a constricted positive column, and on the atomic side the likelihood that the excited oxygen atoms are largely produced in resonant collisions between metastable atoms of argon and normal oxygen molecules. The work emphasizes the fact that the criteria for a tube to be a good spectroscopic source, and to be amenable to electrical analysis, are frequently incompatible.

§ 2. EXPERIMENTAL WORK

(*a*) *Discharge tubes.* McLennan found that the auroral line, although obtainable from positive column discharges through pure oxygen, or mixtures of oxygen with neon or helium, was produced most readily from mixtures of oxygen and argon. We have used this mixture almost exclusively. The argon was obtained from the British Oxygen Company, Ltd., and was purified either in a well-degassed misch-metal arc, or by sputtering tungsten, the second method being found the more efficient. The oxygen was made by heating potassium permanganate in a vacuum. The discharge tubes, which were used connected to the pumping train, were of conventional form (π-shaped, inverted-J, and bulb tubes) in soda glass or pyrex, with the part containing the positive column not less than 1 cm. in diameter. The electrodes for the cold-cathode discharges consisted of heavy plates of tungsten. These were glowed out by induction currents, the tubes were degassed for long periods in a furnace, and mercury and other condensible vapours were kept from them by liquid oxygen traps. The sources of current were accumulators, the 440-v. d.c. mains and a d.c. motor generator capable of giving up to 4000 v. but rarely used above 1000 v. The current was controlled with a wire resistance or a thermionic diode. An attempt to replace the tungsten plate cathode by an oxide-coated thermionic cathode failed, owing to slight persistent contamination of the gas from the oxide coating. The impurities which it was found most necessary to avoid were water vapour, detected by the hydroxyl bands, and carbon compounds. Some specimens of tungsten which were tried acted as continual sources of the latter and had to be discarded.

In accord with the results obtained by McLennan, we have found that in positive-column excitation of the auroral line the oxygen content (which cannot be estimated exactly, owing to electrical clean-up) is not critical, but can be varied from a trace to several per cent, that the best value for the total pressure of the gas is a few millimetres, and that whilst the auroral line appears to be most nearly isolated when the current density is very low, its absolute intensity increases over a large range with increase in current.

(*b*) *Attempts at probe analysis.* The concentration and velocity-distribution of the electrons in an ionized gas can be found under appropriate conditions from the {current, voltage} characteristic curve of a small conducting probe inserted in it. Such information permits of the prediction of the optical properties of a tube if the excitation functions are known, or conversely gives information about the excitation functions from the light emission. We have used probe wires in all the types of tube which have been found to emit the green line. In no case were characteristic curves obtained which could be interpreted. When the logarithm of the electron current to the probe is plotted against the probe potential, the resulting curve should show a discontinuity in slope when the probe potential is the same as that of the surrounding gas. The semi-logarithmic curves were found to be either smooth or too irregular to analyse. It has also been found impossible to locate the space-potential by Langmuir's hot-wire method[13]. Since determination of the space potential is a necessary preliminary to analysis of the curves for electron data, the probe method for finding the internal state of the tubes has had to be abandoned. The failure of the probe method when the current-density is very low is known in other instances, and is probably due to the presence of positive space-charge[4]. Its failure with the tubes passing heavier currents remained unexplained until it was noticed that they contained moving striations, which closer examination showed to be very irregular. We associate the anomalous behaviour of the probes with the irregularity of the striations, since probe characteristics which can be analysed can be obtained with regular striations, although the significance of the data so obtained is uncertain[20].

(*c*) *Moving striations.* A study of the moving striations was made in tubes of inverted-U form. Viewed with a rotating mirror, the bright sections were pink and the intermediate, relatively dark sections green. The reality of the latter colour was however doubtful, as it could be produced by visual contrast. The effect of adding a trace of oxygen to pure argon was striking. The moving striations in pure argon were regularly spaced, with a frequency of 1 to 2 kc./sec. The addition of oxygen made their motion chaotic, and was accompanied by an approximately fourfold increase in frequency, the tube giving out a hissing sound. The effect of oxygen is different from that of mercury vapour, addition of which leaves the striations regular. It seems possible that the irregularity of the striations, which we have found an invariable accompaniment of the emission of the auroral line, is due to fluctuating exchanges of oxygen between the gas and walls. The diffusion of the oxygen through the positive column could be followed by the change in type of the striations. It proved impossible to follow the irregular movement of the striations with the photoelectric shutter used to study their spectra in other gases[20], and the changes in intensity in the auroral line through them could not therefore be traced. Pupp's oscillographic method for probe analysis of moving striations[18] was also inapplicable on account of the irregularity of the movement. McLennan found that the auroral line did not appear at once when the discharge was started, but took a short time to develop. Our observations on the change in type of the striations suggest that this may have been due either to diffusion of oxygen, or to removal of electronegative impurities to the neighbourhood of the anode as negative ions.

(*d*) *Low-voltage arcs.* Druyvesteyn[2] has pointed out that the units of moving striations have some resemblance to low-voltage arcs. It has also been found that the 2270-A. forbidden line of HgI is emitted from a low-voltage arc in mercury vapour[11]. In view of the difficulty of obtaining direct information about the striations in the argon-oxygen mixtures, a low-voltage arc tube was therefore built for use with these. It ran satisfactorily so long as it contained pure argon, but when oxygen was added, even in very small quantity, the filament became partially poisoned, and it was necessary, in order to maintain the discharge, to make the potential-difference between anode and cathode so great that the green band of O_2^+ was excited strongly. This masked the auroral line, if it was present. The filament recovered its original emission when the oxygen had disappeared under the action of the discharge.

(*e*) *Aureoles.* The irregularity of the moving striations, and the failure of the experiment with a low-voltage arc, made it necessary to use less direct methods to find the velocity-distribution of the electrons. It had been noticed at the beginning of our work that when discharges of medium current (e.g. 20 mA.) were passing, the main part of the light from the positive column came from a rosy core, occupying a small fraction of the total cross section of the tube, and that this was surrounded by a green aureole with a diameter several times that of the core. Previous workers have recorded a constriction of the discharge in tubes emitting the auroral line. The light from the aureole has been examined in two ways, both of which show that it is largely due to the auroral line.

In the first method, the middle of a tube 1·6 cm. in diameter was enlarged to a bulb 8 cm. in diameter. A side tube was fused on to the bulb, with its peripheral end blown out to a thin bulb, through which an undistorted view of the centre of the main bulb was obtained. When the discharge was passing, the cores from the narrow sections of the tube projected into the main bulb, and by adjusting the current they could be made almost to meet at the centre. The aureoles near their tips then occupied a large portion of the bulb, and their brightest parts could be examined through the side tube. The light from them, which was still feeble, was always found in this way to come largely from the auroral line. In one case, with a discharge at 800 v. through a mixture of 3 mm. of argon and 0·1 mm. of oxygen, visual observation showed the auroral line alone in the spectrum of the aureole and absent from the spectrum of the core.

In the second method a cylindrical tube was used, with one end blown to a thin viewing bulb, and the electrodes in side tubes. The end-on image of a small section of the positive column was thrown on to the slit of the spectrograph. The light from the aureole then illuminates a greater length of the slit than the light from the core, and gives rise to longer lines in the spectrum. In practice, differences in length are partly obliterated by scattering and instrumental aberrations, by the small movement of the core in the tube, and by the inevitable lack of sharpness of the image of an object viewed longitudinally. These defects, and any effect of variation of plate sensitivity with wave-length when the spectrum is photographed, can be partly overcome by comparing the auroral line with a line of roughly the same intensity in

the same region of the spectrum. By adjusting the discharge conditions, the Ar I line at 5589 A. can be used for reference in this way. Part of a typical spectrum is shown in the plate. The tube current was about 100 mA. and the pressures of argon and oxygen were 6 mm. and about 0·3 mm. respectively. The spectrum was obtained with a glass Littrow instrument with a dispersion of 15 A. per mm. at 5575 A. It is seen that the argon reference line is more concentrated than the auroral line in the spectrum of the core. The apparent spreading of the two strong argon lines is probably mainly instrumental in origin. In the original plate, the auroral line can be traced out to the point marked with a cross, whilst 5589 A. extends about half as far. The difference in their behaviour could also be seen on removing the plate holder from the spectrograph and examining the spectrum in situ with a magnifying glass; it was seen and photographed repeatedly. Other oxygen lines, when present, behaved like argon lines, i.e. came mainly from the core. From these experiments, and those with the bulb tube, it seems likely that the auroral line comes largely if not entirely from the aureole.

(*f*) *Other discharges.* The negative glow is a strong source of Ar II lines, one of which occurs at 5577·7 A. Examination of the spectrum of the negative glow in argon-oxygen mixtures, with discharges from cold tungsten, nickel, and sodium cathodes, showed no enhancement of the Ar II lines such as would be expected if the auroral line were also present. Confirmation of its absence from the light of the negative glow is provided by Sommer's observation[21] that it is not obtained from a condensed positive-column discharge, since the spectra of such discharges are similar to those of negative glows[6]. The auroral line has not been detected in the light from the Faraday dark space, or in the afterglow of a positive-column discharge through argon-oxygen mixtures.

(*g*) *The transauroral line.* From the theoretical transition probabilities, the transauroral line should be detectable when the auroral line is strong. We have succeeded in recording it under these conditions with long exposures, using a Hilger E 39 spectrograph, and photographing through a quartz window attached to the tube by a graded seal. The wave-length calculated from our plates is 2972·3 ± 0·15 A. Our observations do not indicate whether it comes from the core or from the aureole, but since it is emitted in a transition from the same initial level as the auroral line, it probably comes from the aureole.

(*h*) *Continuous spectra.* When the auroral line is present, a continuum passing from the red to the ultra-violet also frequently appears. With discharges of small current-density, this appears to be associated in some way with the auroral line, and is possibly a spectrum of the molecule Ar_3. With discharges of high current-density it comes mainly from the core, and could arise in other ways.

§3. CONDITIONS IN THE DISCHARGE TUBES

(*a*) *Low-current discharges* (*argon-oxygen mixtures*). When the current-density is very small, the positive column is not markedly constricted. McLennan has pointed out that the excitation potentials for the few lines present show that the majority of

the electrons have energy less than some 13 v., and hence that conditions are favourable for the production of metastable argon atoms. It does not seem possible to discriminate between the 1^{s} state, with an excitation potential of 11·49 v., and the 3^{s} state, with an excitation potential of 11·66 v.

In the absence of probe data, it is desirable to obtain some confirmation of McLennan's deduction. Some evidence appears to be provided from the properties of the very low-current discharges studied by Townsend[24]. These are not self-maintained, but conditions in them are probably not greatly different from those in a self-maintained positive column, provided that interaction between the electrons is of secondary importance in the latter compared with collisions between electrons and atoms. Townsend has shown that in such discharges, through pure argon, the mean energy of agitation of the electrons increases from 11·2 v. to no more than 12 v. when the ratio of the field (v./cm.) to the pressure (mm. of mercury) is raised from 2 to 15. From a comparison of the measured and calculated ionization coefficients α/p for electrons in argon, it has been shown that the number of electrons with energy greater than the mean is much less than in a Maxwellian distribution[7]. Provided that addition of a little oxygen does not alter these results materially, it follows that the distribution is probably like that postulated by McLennan, in spite of the fact that the longitudinal field is not necessarily small when the current-density is low[13].

This result is in general accord with the facts that most workers have reported that it is necessary, in order to obtain the auroral line, to have a small longitudinal field, and that when conditions are such that many electrons with greater energy are present, as in a condensed positive-column discharge, the auroral line does not appear.

(*b*) *High-current discharges* (*argon-oxygen mixtures*). Conditions in constricted positive columns have been investigated by Seeliger[22]. The axial concentration of the discharge is due to the establishment of a radial temperature-gradient, assisted probably in the case of argon-oxygen mixtures by the formation of negative ions[8]. Since the average longitudinal field is uniform over the cross section of the tube, the main part of the discharge passes axially, where the density of the gas is least. In our tubes the current-density in the core was of the order of 1 amp./cm.^{2} Seeliger has shown that under such conditions the axial temperature can be several hundred degrees higher than the wall-temperature. Exact calculations of the rise in temperature cannot be made, since the fraction of the energy supplied which is converted into heat in the gas depends on the amount of impurity, here oxygen, added to the main constituent. There is little doubt, however, that in the tubes with constricted positive column emitting the auroral line, the greater part of the current is passing through a hot core of relatively low density embedded in cooler gas of higher density. Seeliger has also found that a molecular gas present as impurity tends to concentrate outside the core. In the present case this will tend to result in the oxygen being atomic in the core and molecular elsewhere.

The properties of the cored discharge suggest two possibilities for explaining why excitation of the auroral line occurs in the aureole. The first follows from a general

theory given by Seeliger to account for differences between the spectra of cores and aureoles. The core is a plasma, with high ionization, in which the velocity-distribution of the electrons is largely determined by interaction between the charged particles, and will probably be nearly Maxwellian. In the aureole, since the density of the gas is higher and the current-density much less than in the core, the velocity-distribution of the electrons will be determined more by collisions between the electrons and atoms; that is to say, conditions will be similar to those in low-current discharge tubes. The second follows from the work of Duffendack and Smith[3] and others on the propagation of resonance radiation in the inert gases. It has been found that a positive column emits resonance radiation, which can be absorbed elsewhere in the gas and produce metastable atoms of the inert gas by secondary reactions. In cored discharges, resonance radiation emitted from the core could be absorbed in the aureole, and could produce metastable atoms in this way. The final result in the aureole is the same as if a feeble current were passing.

Other possible effects of the core on the aureole include (1) diffusion of positive ions and electrons into the aureole, which recombine there—this should lead to an enhancement of high series lines of Ar I in the aureole, which we have not observed; (2) diffusion of metastable atoms, if present in the core, into the aureole; (3) illumination of the aureole with Ar I radiation from the core, which is absorbed by the metastable atoms and reduces their concentration.

(*c*) *Discharges through other mixtures, and through pure oxygen.* These have not been studied in the same detail as discharges through argon-oxygen mixtures. In general they are less strong sources of the auroral line. Presumably in them resonant collisions of the second kind cannot occur with the same probability as in argon-oxygen mixtures (§ 4), and the excitation is largely a result of electron collisions with oxygen atoms and molecules, with the velocity-distribution of the electrons not particularly favourable for direct or indirect excitation of the ^1S state of the atom.

§ 4. ATOMIC PROCESSES IN PRESENCE OF ARGON

The preceding discussion of excitation conditions, although not conclusive, indicates that one or both of two factors may be primarily involved. There may be present numerous electrons with energy of the order of 12 V., or metastable argon atoms. The latter may be produced by the electrons, or by resonance radiation. When both metastable argon atoms and electrons with this restricted range of energies are present, it is impossible to decide the extent to which each is responsible for the activation of the oxygen, without greater knowledge of the terms and excitation functions of the molecular oxygen spectrum than is at present available. Direct collisions between electrons and oxygen molecules, to whatever extent they occur, will be capable of description in the same way—in accordance with equation (*b*) below—as collisions between metastable argon atoms and oxygen, with the substitution for the argon atom of an electron of the same or slightly greater energy. From considerations of energy alone, metastable argon atoms could excite oxygen atoms in the ground state to the ^1S metastable state in collisions of the second kind.

The weight of evidence goes to show however that whatever process occurs is efficient, and hence probably involves quantal resonance. This is not compatible with the energy discrepancy of 7·4 v. in the reaction

$$Ar' (1° \text{ or } 3°) + O (^3P) \rightarrow Ar (^1S) + O' (^1S). \qquad (a)$$

Exact resonance would give excitation to 16701 cm.$^{-1}$ in O I, from 1° metastable Ar', or 15291 cm.$^{-1}$ from 3° metastable Ar'. The nearest terms in O I are $2p^3$ (^4S) $3p\,^3P$ 21207 cm.$^{-1}$, and $2p^3$ (^4S) $4s^5$ S, 14358 cm.$^{-1}$. Excitation to either of these, followed by subsequent transitions to the ^1S state, again involves too great an energy discrepancy to be very likely. The primary collision is therefore probably one between a metastable argon atom and an oxygen molecule in its lowest electronic state ($x^3\Sigma$), the oxygen atoms in the ^1S state being formed by spontaneous dissociation of the resulting excited molecule. The choice of excited molecular states is limited by the conditions that the energy must not be greater than that of metastable argon, and not less than the sum of the energy of dissociation of normal ($x^3\Sigma$) molecules into two ^3P atoms, and the energy of excitation of a ^1S atom, or 9·26 v. These conditions can be satisfied only for those states which dissociate into one ^3P and one ^1S atom, with asymptotic energy 9·26 v., and those dissociating into one ^1S atom and one ^1D atom, with asymptotic energy 11·22 v. The processes which might lead to production of the auroral and transauroral line can then be summarized as follows:

$$Ar' (1° \text{ or } 3°) + O_2 (x^3\Sigma) \rightarrow Ar (^1S) + O_2' (?) \qquad (b)$$

$$O_2' (?) \xrightarrow{\text{or}} \begin{cases} \rightarrow O' (^1S) + O' (^1D) + 0{\cdot}3 \text{ v. k.e.} & (c) \\ \rightarrow O' (^1S) + O (^3P) + 2{\cdot}2 \text{ v. k.e.} & (c') \end{cases}$$

$$O' (^1S) \begin{cases} \xrightarrow{90\,\%} O' (^1D) + h\nu\,(5577 \text{ A.}) & (d) \\ \xrightarrow{10\,\%} O (^3P) + h\nu\,(2972 \text{ A.}) & (d') \end{cases}$$

Considering the processes in more detail, the first question which arises is whether it is legitimate to regard as distinct the excitation (*b*) and the dissociation (*c*). It cannot be rigorously legitimate, but if there is resonance the exchange of energy may occur for separations much greater than those which correspond to gas-kinetic collisions. This implies that the internuclear distance in O_2 will remain approximately constant in the collision. There is some evidence that this occurs in collisions between various molecules and metastable helium atoms[5]. The process is also similar to that which is supposed to occur in the excitation of the Lyman bands of hydrogen in presence of argon, the chief difference being that the excited hydrogen molecule which is formed is stable as regards dissociation, and loses its energy in a reversal of the excitation process with emission of radiation[23]. A second question is that of the nature of the excited state or states produced. These might be of $^1\Sigma$, $^1\Pi$ or $^1\Delta$ type (11·22 v. asymptotic energy), or of $^3\Sigma$ or $^3\Pi$ type (9·26 v. asymptotic energy), and might have a potential-minimum or be repulsive at all internuclear distances. The molecular spectrum of oxygen is not sufficiently well known for a decision to be made between them on the assumption that the internuclear distance is unaltered, and it is also impossible to obtain any assistance from the principle of

the conservation of spin, as the possible vectorial combinations are too numerous. It is, however, known that molecular states of the type required exist in the neighbourhood of 11 v., since the molecular absorption spectrum shows anomalies between 1000 A. and 1200 A.[17] The kinetic energy liberated in (*c*) will lead to no appreciable broadening of the lines, as the metastable oxygen atoms will be practically in thermal equilibrium with the surrounding gas before they radiate.

The sequence of processes (*b*), (*c*) and (*d*) throws some light on the inhibitory effect which some impurities have on the production of the auroral line. No emission can take place if any one of the processes is prevented from proceeding as indicated. Inhibition of dissociation need not be considered, as it will take place rapidly when once the excited molecule has been formed. A collision of the second kind between an impurity molecule and either a metastable Ar′ or O′ atom may result in interruption of the sequence. In the case of collisions with Ar′, the impurities compete with the normal oxygen molecules, and the amount which can be tolerated will be determined by the amount of oxygen present and by the closeness of resonance in the process, similar to (*b*), by which the impurity becomes excited by the Ar′. The same remark applies to collisions of the second kind between impurities and O′, but these may be expected to be less important than collisions with Ar′, since most molecules likely to be present are not rich in electronic levels so close as 4·2 v. to their ground state. A chemical reaction between the O′ and impurity can, however, occur. Both Ar′ and O′ atoms can also undergo transitions in collisions of the first and second kinds with electrons. These cannot be discussed quantitatively in the absence of probe data for the electrons. Qualitatively, neither effect would be expected to be of importance in the aureole, but either might be important in the core. It is possible that destruction of metastable atoms in these ways by electrons is responsible for the weakness or absence of the auroral line from the light of the core.

It has been assumed that the emission of the auroral line is spontaneous. This is probable, both because it can be obtained from feeble discharges, and because it is narrow. On this assumption it is possible to make a very rough estimate of the concentration of O′ atoms in the ^1S state. In a discharge such as that which gave the spectrum shown in the plate, the total amount of light reaching the slit is about the same for the auroral line and for Ar I 5589 A. The latter is emitted, from the core, in a transition from the $5s_1''''$ level, with an excitation potential of 15·2 v. If the concentrations of the excited oxygen and argon atoms are n_0 and n_A, the transition probabilities A_0 and A_A, and the lengths along the tube radius for which the lines are emitted r_0 and r_A, then $n_0 A_0 r_0$ and $n_A A_A r_A$ must be approximately the same. Since the core is highly ionized, n_A may be related to the concentration N of unexcited Ar atoms by a Boltzmann factor involving the electron temperature W[12], or if the statistical weights are neglected,

$$n_A = Ne^{-11.2/W},$$

W will be of the order of unity and N of the order of 10^{17} cm.$^{-3}$, and hence n_A is about 10^7 cm.$^{-3}$. If $A_0 = 2$ and A_A is estimated to be $5 \cdot 10^8$ while $r_0/r_A = 10$, n_0 is $2{\cdot}5 \times 10^{12}$ cm.$^{-3}$ If the total concentration of the oxygen were five per cent of that of the argon, about

$5 \cdot 10^{-2}$ per cent of it would be in the ^1S state. These calculations are very uncertain, on account of the entry of W into the exponential factor, but it seems impossible, from the relative values of A for permitted and forbidden transitions, to escape from the conclusion that the concentration of metastable oxygen atoms is large.

§ 5. GENERAL CONSIDERATIONS

The consideration of electrical and atomic processes in the two preceding sections does not exhaust all possible factors which may be involved in the production of the auroral and transauroral lines. It is difficult to be sure that even all major effects have been taken into account when one is dealing with systems so complex as these discharges. It is however interesting to consider whether any conclusions about the production of forbidden lines in general can be reached on the basis of the results which we have obtained, and this wider problem can be approached by examining why the forbidden red lines of oxygen, in particular the strongest line (6300 A.), were not emitted from the tubes which we have used. Oxygen atoms in the initial (^1D) state for this transition can be produced from oxygen molecules by metastable argon atoms in the processes (*c*) and (*d*). They may also possibly be formed by a third process, in which an excited molecule is formed which dissociates into two atoms in the ^1D state, with asymptotic energy 9·0 v. The rate at which atoms are formed in the ^1D state may thus be somewhat greater than the rate at which they are formed in the ^1S state. The intensity of the red line depends, however, not on the rate of formation of atoms, but on the stationary concentration n set up, particularly near the core, and it can be seen that the integral of An throughout the accessible parts of a tube of ordinary dimensions should be less than the corresponding integral for the auroral line. The difference arises from the fact that A is 270 times greater for the auroral line than for the red line. This operates in two ways. Firstly, a metastable ^1D atom is much more liable to undergo change by a collision during its lifetime than a ^1S atom, and secondly, even if no deactivating collision occurs, it may be lost by diffusion to the walls or elsewhere. An approximate treatment of the effects of diffusion can be given by taking the mean distance ξ to which an oxygen atom diffuses before radiating to be of order given by

$$\xi = 2\,(D/A)^{\frac{1}{2}},$$

where D is the diffusion coefficient. D is unknown for these metastable atoms, but from the values of other diffusion coefficients[10] will probably be about $0 \cdot 3\alpha^2/\beta$, where α is the ratio of the temperature to 273°, and β the ratio of the pressure to 760 mm. For the green line, $A = 2$ sec.$^{-1}$, and with typical values of 2 and 3/760 for α and β respectively, $\xi = 25$ cm. This is rather large compared with the diameter of the core, but not much greater than the diameter of the bulb in the tube referred to in § 2, the whole of which was sometimes seen to emit the green line. For the red line, ξ is some 16 times longer. It would therefore be expected to be observable with any ease only with wider tubes than we have employed, or with cored discharges of higher current density. One or both of these conditions seem in fact to have been observed by previous workers who have obtained it.

In the case of forbidden lines of the ions of the light elements, conditions become even more unfavourable. The metastable excited ions will, in addition to normal diffusion forces, be also subject to radial electric fields, tending to draw them towards the walls. They are also liable to be neutralized by electron-exchange in collisions with other atoms and molecules, which is probably a main cause of their destruction. We were unable to obtain forbidden lines of the nitrogen and oxygen ions by passing an intense condensed spark through the edge of an arc, or the forbidden lines of neon by passing a condensed spark through a negative glow, or by passing a high voltage spark through the gas. The first method can apparently be applied successfully if still heavier discharges are used[16]. The full conditions for success in these cases are not clear, but the experiments on the auroral line indicate that they may include the establishment of an intensely ionized core, which maintains an aureole, and that this can act as a source of the forbidden lines when certain auxiliary conditions are satisfied.

§6. ACKNOWLEDGEMENTS

In the preparation of this paper, use has been made of the note-books and theses of M. P. Quinlivan (1932–3), M. K. Bell and J. Malone (1932–3), and C. M. Minnis (1933–6), to whom many of the ideas and results presented are due. We are also indebted to the Government Grant Committee of the Royal Society for providing part of the apparatus which we have used.

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Ultra-violet band system of CSe (quartz prism spectrogram)

To face page 989

A REINVESTIGATION OF THE ULTRA-VIOLET BAND SYSTEM OF CARBON MONOSELENIDE

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Communicated by W. Jevons 28 *July* 1939

ABSTRACT. The ultra-violet band system of CSe observed by Rosen and Désirant has been produced in a heavy-current uncondensed discharge through a silica tube into which small quantities of a mixture of aluminium, selenium and a hydrocarbon grease are introduced. The accuracy of the derived constants is considerably increased, as a result of measurements of some twelve band-heads in the region $\lambda 2760-\lambda 3054$. With the exception of band-heads affected by perturbations associated with $v'=1$, the data are well represented by

$$\nu_{\text{head}}=35238{\cdot}0+(835{\cdot}7u'-2{\cdot}2u'^2)-(1036{\cdot}0u''-4{\cdot}8u''^2)-1{\cdot}3u'u'',$$

where $u=v+\frac{1}{2}$. The vibrational coefficients in this expression, and the estimated value, 6·8 ev., of the energy of dissociation of the ground state of CSe, are brought into satisfactory relation to those of the neighbouring molecules CO, CS and SiSe. Unsuccessful attempts have been made to develop the corresponding system of CTe.

In the course of a survey of corresponding band systems of the oxides, sulphides, selenides and tellurides of the elements of group IV-*b*[1,2] the existing data for these molecules were examined critically. Those referring to the ultra-violet system of CSe were found to be particularly unsatisfactory, in so far as the only available observations, namely, Rosen and Désirant's[3], include only ten bands of the system, obtained in emission, and their analysis is based on measurements claimed to be accurate, in the case of the strong bands, to within only ± 5 cm.$^{-1}$; their approximate expression for the observed band-heads is

$$\nu_{\text{head}}=35250+(830u'-?)-(1037u''-8u''^2),$$

where $u=v+\frac{1}{2}$. The present note gives the results of a reinvestigation of this system, in which these authors' analysis has been confirmed as a result of more accurate observations of these bands, obtained in emission, and photographed on instruments giving dispersions two and four times greater than that used by Rosen and Désirant.

The source used for the production of the system was a heavy-current discharge through a silica tube of the type shown in figure 3 of a previous paper[4], small pellets of powdered aluminium and selenium bound with a hydrocarbon grease being introduced into the positive column as and when required. Current for the discharge tube was taken from a 2400-v. 5kva. transformer connected to the 230-v. supply of a.c. at 50 c./sec: the optimum value of the current through the tube was found to be about 1·5 amp. The system was photographed against Fe arc lines on

Ilford Ordinary and Zenith 650 plates in (i) a first order of a 2·4-m. concave grating in an Eagle mounting with dispersion of about 7·42 A./mm., and (ii) a large quartz Littrow instrument whose dispersion varied between 2·40 A./mm. at λ2600 and 3·90 A./mm. at λ3000. Exposures were of the order of six minutes. A reproduction of a prism spectrogram is given in the plate.

The system consists of red-degraded bands extending from λ2760 to λ3053, the 0→0 band being near λ2844·0. Atomic lines present are due to C, Si, Ni, Pb, Sb, Tl, Ag and Ge: other bands developed are those of CS on the short-λ side and OH at λ3064, which probably curtails observations on the long-λ side of the CSe system. The general appearance and intensity-distribution in this system are, as Rosen and Désirant remarked, very similar to those observed in that of CS, as excited by an analogous method.

Measurements of the band-heads and the assigned values of the vibrational quantum numbers are set out in a Deslandres scheme in table 1. The equation

$$\nu_{\text{head}} = 35238{\cdot}0 + (835{\cdot}7u' - 2{\cdot}2u'^2) - (1036{\cdot}0u'' - 4{\cdot}8u''^2) - 1{\cdot}3u'u''$$

represents well the measurements on the best observed bands, and involves the assumption of a perturbation of the order of 10 cm.$^{-1}$ in the $v' = 1$ level. The

Table 1. Deslandres' arrangement of band-head data.

Italics denote wave-lengths in air (I.A.); large Roman, wave-number *in vacuo* (cm.$^{-1}$); small Roman, wave-number differences *in vacuo* (cm.$^{-1}$).

v'	v'' → 0		1		2		3		4
3					*2810·60*				
					35569·1				
					819·9				
2			*2794·93*		*2876·92*		*2963·51*		*3054·21**
			35768·6	1019·4	34749·2	1015·3	33733·9	1001·7	32732·2
			835·8		834·0		833·6		
1	*2779·98*		*2861·80*		*2947·67*		*3038·58*		
	35960·9	1028·1	34932·8	1017·6	33915·2	1014·6	32900·6		
	821·7		819·5		818·7				
0	*2844·99*		*2930·55*		*3020·59*				
↑	35139·2	1025·9	34113·3	1016·8	33096·5				

* Measure probably upset by OH structure.

necessity of the term in $u'u''$ is seen in table 1 in the systematic increase (i) of each of the vibrational intervals $G'(\frac{1}{2})$ and $G'(1\frac{1}{2})$ with diminishing values of v'', and (ii) of each of the vibrational intervals $G''(\frac{1}{2})$, $G''(1\frac{1}{2})$ and $G''(2\frac{1}{2})$ with increasing values of v'[4]. This term, which takes account of the variation of the intervals between R-head and band-origin from band to band, is to be expected in the case of a system such as this in which the distribution of band intensity is of the type represented by a narrow Condon parabola.

Besides the bands listed in table 1, the discharge is invariably accompanied by bands at λ2761·76 and λ3010·55 which may be the 5→3 and 5→6 bands of the present system, and by a band at λ2884·53, which remains unidentified.

The vibrational constants and system origin will be discussed in their relation to the data for corresponding systems in a forthcoming paper[2] describing band

systems of GeSe and GeTe; here it will only be noted that the value for the energy of dissociation in the ground state obtained by extrapolation by means of the expression

$$D = (\omega_e - x_e\omega_e)^2/(4x_e\omega_e \times 8106),$$

viz. 6·8 ev., although probably too high, is in satisfactory relation to the values, 9·1, 7·8 and 5·8 ev. for the ground states of CO, CS and SiSe[5] respectively.

In conclusion, it may be mentioned that attempts have been made to obtain the corresponding system of CTe by analogous methods under a variety of experimental conditions in which carbon and tellurium have manifested their simultaneous presence in the positive column of the discharge tube: on certain of the plates it is possible to locate a few weak bands in the expected region, but they are inadequate for a vibrational analysis, nor is it even certain that they belong to the CTe system.

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THE EXAMINATION OF DENTAL ENAMEL BY X RAYS AND THE POLARIZING MICROSCOPE

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Received 23 *May* 1939

ABSTRACT. The phenomena of birefringence exhibited by human dental enamel are shown to be consistent with the crystalline arrangement previously suggested by the author on the basis of x-ray measurements. The main feature of this arrangement is the presence of two groups of apatite crystallites within each enamel prism. The hexagonal axes of one group are inclined to the prism direction at about 5°, and those of the other group at about 40°, the 5° group usually being predominant. x-ray examination suggests that both groups of crystallites are also present in the interprismatic substance, and the optical properties of this material indicate that it contains, in general, a majority of the 40° crystallites.

§ 1. INTRODUCTION

In a recent paper[1] the author has published the results of an x-ray investigation into the submicroscopic structure of dental enamel, in which it is shown that an enamel prism contains a multitude of regularly arranged apatite crystallites, the orientation of which is worked out in detail for human deciduous enamel. It is clear that if enamel consists of regularly arranged crystallites, examination with the polarizing microscope should give useful information; and much important work on this subject has in fact been carried out by W. J. Schmidt, and by his pupils A. Keil and M. Harders-Steinhäuser. This work, although dealing to a large extent with problems not attacked by x-ray methods, has also led to certain conclusions regarding the nature of enamel, which can be examined in the light of x-ray evidence; and it is the purpose of the present paper to attempt to correlate the optical and x-ray results.

It may perhaps be desirable, at this stage, to describe very briefly a few of the main facts concerning the histology of human dental enamel. This material is by no means homogeneous and microscopic examination reveals that it is largely made up of long narrow rods which are often hexagonal in cross section and are therefore frequently referred to as "prisms". The breadth of a prism is usually between 2×10^{-4} cm. and 5×10^{-4} cm. and each prism runs from the enamel-dentine boundary to the outer enamel surface. The prism material is mostly inorganic, but each prism is enclosed by a very thin sheath of organic material. The prisms, with their organic sheaths, are separated from each other by the interprismatic substance,

which again is mostly inorganic. The directions in which the prisms run in a tooth are shown in figure 1.

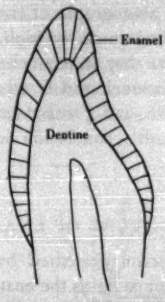

Figure 1. Diagram showing the directions of the enamel prisms in an average tooth.
(From Orban's *Dental Histology and Embryology*.)

§ 2. THE COMPOSITION OF ENAMEL

When tooth sections are examined with polarized light the enamel is clearly seen to be birefringent, and this was recognized as long ago as 1861 by Valentin[2].

The birefringence of a substance may be due either to the intrinsic birefringence of its crystalline constituents (*Eigendoppelbrechung*) or to a regular arrangement within the substance of rod-shaped particles, of which the thickness and distance apart are small compared with the wave-length of light (*Formdoppelbrechung*). Unless the refractive index of these particles is different from that of the medium in which they are contained, however, the latter type of birefringence will not be observed.

The theoretical aspect of form birefringence was dealt with by Wiener[3], and form birefringence and eigen birefringence were both observed by Ambronn in alumina[4], and in celloidin and cellulose[5]. It was left to W. J. Schmidt, however, to show that form birefringence and eigen birefringence could also be observed together in developing human enamel. He concluded[6, 7, 8] that this type of enamel is "built up of small rod-shaped particles, arranged so that their lengths are more or less exactly parallel to each other and to the optic axis—and therefore also approximately to the prism axis". Schmidt also confirmed the view of v. Ebner that the further calcification of enamel takes place by the deposition of new calcific particles between those already laid down, and came to the conclusion that an enamel prism is composed of submicroscopic particles of a uniaxial crystalline substance similar to apatite.

Chemical analysis indicates[9] that enamel does in fact consist largely of hydroxy-apatite, $Ca_{10}(PO_4)_6(OH)_2$, and this conclusion is fully confirmed by x-ray analysis (see, for example, Thewlis and others[10]).

The size of the crystalline particles in enamel has been estimated by Möller and Trömel[11] to be 10^{-4} cm. and by Bale and others[12] to be about 3×10^{-5} cm. Examination of X-ray diffraction photographs of enamel shows that the latter size is of the right order, the former being much too high.

This being the case, it is clear that an enamel prism, of which the length is at least equal to the depth of the enamel, and of which the breadth is, as has been stated above, between 2×10^{-4} cm. and 5×10^{-4} cm., must contain a multitude of apatite crystallites. Thus Schmidt's conclusion is so far consistent with X-ray evidence.

§ 3. THE STRUCTURE OF ENAMEL PRISMS

The picture of an enamel prism presented by the polarizing microscope is confirmed by X-ray examination in so far as the enamel is shown to consist of small crystalline particles of apatite. The optical results suggest, in addition, that the optic axes of these particles are approximately parallel to each other and to the prism direction. But according to the author's X-ray measurements, although the axes of some of these particles are approximately parallel to the prism direction, being inclined at about 5° to it, those of other particles are inclined to it at about 40°. These axes point to that side of the prism which is remote from the tip of the nearest cusp, the directions being taken away from the tooth as in figure 2.

At first sight the X-ray and optical results do not appear to agree, but it is shown below that the optical properties of a system of crystalline particles arranged as in figure 2 are in fact similar to those which are exhibited by enamel prisms.

If a longitudinal section of a tooth be cut so that the enamel prisms lie in the plane of the section, and if this section be examined with polarized light between crossed nicols, then figure 2 will represent the relationships between a prism, its constituent crystallites, and the directions of vibration in the nicols.

In this diagram OP represents the direction of the prism at the point O in the enamel, OF_1 and OF_2 the directions to which the optic axes of the two sets of crystallites in the prism are parallel, Op the vibration direction of the polarizer and Oa that of the analyser. The angles POF_1 and POF_2 are 5° and 40° respectively.

Now in the case of a section of a human tooth, optical extinction is usually stated to occur when the direction of vibration in the polarizer or analyser is approximately parallel to the direction of the prism. Further, as Schmidt has shown, transverse sections of prisms remain dark for all settings of the crossed nicols.

This general description, which is true so far as it goes, does not make clear the fact, noted by v. Ebner[13], that in the extinction position, for longitudinal sections, the direction of vibration in the appropriate nicol is never quite coincident with the direction of the prism, but is inclined to it at angles ranging from 5 to 20°. v. Ebner found this not only in sections of teeth but in isolated prisms, so that possible errors due to underlying prisms were excluded. The phenomenon is mentioned by Kitchin[14] and was indeed noticed by Cape and Kitchin[15]. Schmidt has also observed the effect, and it is referred to in the admirable survey of the optical

examination of dental enamel given by Harders-Steinhäuser[16]. None of these workers, however, appears to have stated whether the optic axis is sometimes inclined to the direction of the prism on one side and sometimes on the other, or whether it is restricted to one side only. An optical examination has therefore been made of a series of longitudinal sections of human teeth, and it has been found that extinction only takes place when the direction of vibration in either nicol is inclined to the direction of the prism on the same side as OF_1 and OF_2 in figure 2.

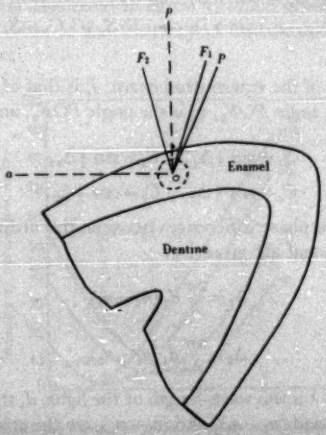

Figure 2. Diagram of section of tooth showing relationship between direction of prism, crystallite axes and directions of vibration of crossed nicols. OP, prism direction; OF_1, direction of crystallite axes (1); OF_2, direction of crystallite axes (2); Op, vibration direction of polarizer; Oa, vibration direction of analyser.

To decide whether or not the optical and X-ray results are mutually consistent, the extinction phenomena, if any, associated with the arrangement of crystallites illustrated in figure 2 will now be determined. In order to do this the intensity of the light transmitted through longitudinal and transverse sections, as the nicols are rotated with respect to the section, will be calculated.

(i) *Longitudinal sections.* The extinction phenomena, if they exist, will be repeated every 90° during rotation, so that we need only consider variations in intensity which take place as the polarizer and analyser are rotated through 90° from any given position. This position is chosen to be that in which the direction of vibration in the polarizer is parallel to the direction of the prism.

The problem is essentially that of calculating the interference effects due to the superposition of two uniaxial crystal plates. The expression for the intensity of the transmitted beam in this case was worked out by Fresnel[17] and has more recently been discussed by Schulz[18].

In the general case the intensity is dependent on the order in which the two crystal plates are struck by the incident beam of polarized light, but for crossed or parallel nicols the intensity is independent of this factor. If it be assumed for the moment that figure 2 represents the arrangement, not of two sets of crystals, but of two crystals only, the optic axis of crystal 1 being parallel to OF_1 and that of crystal 2 being parallel to OF_2, then the expression for the intensity of the transmitted beam, in the case of crossed nicols, reduces to:

$$I = I_0 [\sin^2 2(\alpha-\eta_1) S_1^2 + \sin^2 2(\alpha-\eta_2) S_2^2 + 2 \sin 2(\alpha-\eta_1) \sin 2(\alpha-\eta_2) S_1 S_2 \{C_1 C_2 - S_1 S_2 \cos 2(\eta_2-\eta_1)\}], \quad \ldots\ldots(1)$$

where I is the intensity of the transmitted beam, I_0 is that of the incident beam, α is the angle POp, η_1 is the angle POF_1, η_2 is the angle POF_2, and S_1, S_2, C_1 and C_2 are given by:

$$S_1 = \sin \tfrac{1}{2}\Delta_1, \quad S_2 = \sin \tfrac{1}{2}\Delta_2,$$
$$C_1 = \cos \tfrac{1}{2}\Delta_1, \quad C_2 = \cos \tfrac{1}{2}\Delta_2,$$

where Δ_1 and Δ_2 are the phase differences between the ordinary and extraordinary rays in crystals 1 and 2 and are given by:

$$\Delta_1 = \frac{2\pi}{\lambda} d_1 (n_1-n_2)_1$$

and

$$\Delta_2 = \frac{2\pi}{\lambda} d_2 (n_1-n_2)_2.$$

In these expressions λ is the wave-length of the light, d_1 the thickness of crystal 1 and d_2 that of crystal 2, and $(n_1-n_2)_1$ and $(n_1-n_2)_2$ are the strengths of birefringence of crystals 1 and 2 respectively. If it may be assumed that the two crystals do not differ appreciably in composition, we may write

$$(n_1-n_2)_1 = (n_1-n_2)_2.$$

In order to determine what happens as the polarizer is rotated through 90° from the position in which its direction of vibration is parallel to the direction of the prism, the above expression for I must be evaluated for values of α from 0 to 90°, the values of S_1, S_2, C_1 and C_2 for the particular cases considered being first calculated. This expression only applies to two of the many crystals in a prism; but since the interference effects are additive it may be extended to the case of a tooth section of known thickness D, if the prisms are presumed to be parallel to each other, by replacing d_1 by D_1 and d_2 by D_2, where

$$D_1 + D_2 = D,$$

D_1 and D_2 being proportional to the numbers of crystallites with optic axes parallel to OF_1 and OF_2 respectively.

According to the x-ray results, the first group of crystallites (i.e. crystallites with optic axes parallel to OF_1) usually predominates. The crystallites of the second group may occasionally be as numerous as those of the first, but they are sometimes

absent altogether. In other words, the value of D_1 may vary from $\frac{1}{2}D$ to D; i.e. the percentage of the first group of crystallites may vary from 50 to 100.

The value of I has therefore been calculated for the cases in which $D_1=0{\cdot}5D$, $D_1=0{\cdot}6D$, $D_1=0{\cdot}8D$ and $D_1=D$, corresponding respectively to the presence in the prism of 50, 60, 80 and 100 per cent of the first group of crystallites. The values of D chosen were 20, 50 and 100μ, and the wave-length of the light was taken as 5500 A. The value of (n_1-n_2) used was that given by Keil[19], namely 0·0029.

For all proportions of crystallites considered, it was found that intensity minima occurred. This is illustrated in figure 3, which gives the results for sections of thickness 50μ, the value of the intensity at the maximum (not shown in the figure) being put at 100.

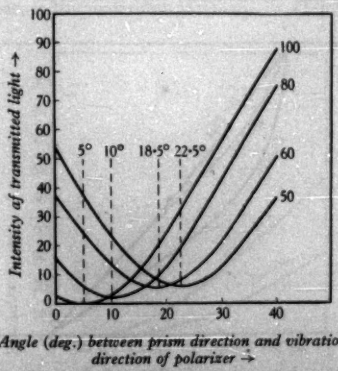

Figure 3. Curves showing variation of intensity of transmitted light as crossed nicols are rotated, for different percentages of first group of crystallites. $D=50\mu$.

It will be seen from figure 3 that as the percentage of the first group of crystallites rises the value of α corresponding to the minimum falls, and the extinction becomes more nearly total. Figure 4, in which the extinction values of α are plotted against the percentage of the first group of crystallites, shows that the value of α for which extinction takes place is also dependent on the thickness of the section.

From the above discussion it will be seen that the arrangement of crystallites suggested by X-ray examination, and illustrated in figure 2, should give rise to extinction in a longitudinal section when the direction of vibration in the polarizer or analyser is inclined to the direction of the prism, on the side remote from the tip of the nearest cusp (the directions being taken away from the tooth), at angles ranging from 5 to 22·5°, according to the relative proportions of the two sets of crystallites present. By the use of the polarizing microscope, as has already been mentioned, it is found that extinction is in fact observed at angles ranging from 5 to 20°.

(ii) *Transverse sections.* In transverse sections the light is travelling along the direction of the prism and not at right angles to it; it is travelling at 5° to the optic axes of the first group of crystallites and at 40° to those of the second group.

Now calculation shows that the birefringence is negligible for light travelling at 5° to the optic axis of apatite and is 0·0015 for light travelling at 40° to this direction.* Consequently if f represents the fraction of the second group of crystallites present in the prism, and D represents the thickness of the section, the effective thickness, as far as birefringence is concerned, will be represented by fD; and the problem of calculating the intensity of the light transmitted along the direction of the prism is reduced to that of determining the intensity transmitted through a single crystal plate, of thickness fD, in such a direction that the difference between the refractive indices of the ordinary and extraordinary rays is 0·0015.

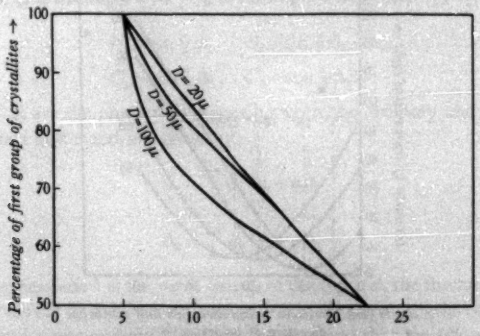

Figure 4. Curves showing relationship between extinction position and percentage of first group of crystallites, for different thicknesses of section.

The expression for the intensity of the transmitted beam in this case, for crossed nicols, is

$$I = I_0 \sin^2 2\phi \sin^2 \tfrac{1}{2}\Delta, \qquad \text{.....(2)}$$

where I is the required intensity, I_0 the intensity of the incident beam, ϕ the angle between the vibration direction of the polarizer (or analyser) and either of the vibration directions of the crystal, and

$$\Delta = \frac{2\pi}{\lambda} d\,(n_1 - n_2),$$

where λ is the wave-length of the light, d the thickness of the crystal plate and $(n_1 - n_2)$ the birefringence. As we know, $d = fD$ and $(n_1 - n_2) = 0{\cdot}0015$.

* The calculation is carried out by determining the lengths of the appropriate radii of the indicatrix.

Clearly when $\phi = 0°$ or $90°$ the value of I will be zero; total extinction will occur. The maximum value of I will occur when $\phi = 45°$, and will be given by

$$I_{max.} = I_0 \sin^2 \tfrac{1}{2}\Delta. \qquad \text{.....(3)}$$

According to the x-ray results a usual value for f appears to be about 0·2, and when this value is substituted in the above expression, the same value for λ being chosen as before, it is found that

$$I_{max.} = 0{\cdot}007 I_0 \text{ for sections of thickness } 50\,\mu,$$

and that $$I_{max.} = 0{\cdot}03 I_0 \text{ for sections of thickness } 100\,\mu.$$

These values do not, of course, give much idea of the brightness at the maximum unless a standard of intensity is available for comparison. A suitable standard, for any particular thickness, is the maximum intensity, in a longitudinal section of the

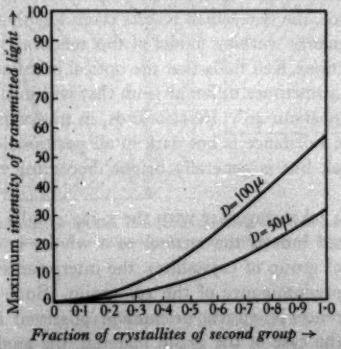

Figure 5. Curves showing relationship between *maximum* intensity of light transmitted in transverse section and fraction of crystallites of second group, for different thicknesses of section.

same thickness, of the light transmitted by prisms composed entirely of one group of crystallites. The value I_s of this intensity is given by equation (3) if we put $d = D$ and $(n_1 - n_2) = 0{\cdot}0029$, and it is then found that

$$I_s = 0{\cdot}54 I_0 \text{ for a section of thickness } 50\,\mu,$$

and $$I_s = 0{\cdot}99 I_0 \text{ for a section of thickness } 100\,\mu.$$

Thus for sections of thickness 50 and 100 μ the maximum intensity in a transverse section is only 1·3 and 3 per cent respectively of the standard intensity.

In figure 5 the relationship between f and the maximum intensity transmitted in a transverse section is shown for sections of thickness 50 and 100 μ, the standard intensity being put at 100 in each case. From the curves in this figure it will be seen that where the first group of crystallites is predominant the prisms should appear dark or nearly so in all positions of the crossed nicols. This, as was mentioned above, is what is observed experimentally.

§ 4. THE STRUCTURE OF THE INTERPRISMATIC SUBSTANCE

It is generally recognized that the interprismatic substance is mostly inorganic in character, and Keil[19] has shown that it is optically birefringent and crystalline. He has also made certain observations, dealt with below, for which he has put forward what is admittedly a tentative and not wholly satisfactory explanation.

The x-ray examination of enamel led the author[1] to the conclusion that "no type of orientation is present in the interprismatic substance which is not also present in the prism material", and it was further concluded that "the relative extent to which the 5 and 40° groups are present in the two cases may...not be identical".

Although neither Keil's results nor those obtained by the author appear in themselves to be capable of giving a satisfactory account of the structure of the interprismatic substance, the two sets of results taken together furnish, as will now be seen, a fairly satisfactory working model of this material.

In longitudinal sections Keil finds that the optical orientation within the interprismatic substance is sometimes different from that within the prisms, and in one case deviates from it by about 40°. He also finds, in transverse sections of prisms, that the interprismatic substance is not dark in all positions of the crossed nicols, like the prism material, but is generally bright, becoming dark only for certain settings.

These observations taken together with the x-ray results suggest that whereas the prism material, and indeed the enamel as a whole, is characterized by the predominance of the 5° group of crystallites, the interprismatic substance may be characterized by the predominance of the 40° group. Such an arrangement will account for the difference in optical orientation observed by Keil; for, as has already been seen in the case of the prism material, the angle of extinction, in a longitudinal section, depends on the proportion of crystallites belonging to each group. Indeed, figures 3 and 4, given in the previous section for the prism material, will be directly applicable, *mutatis mutandis*, to the interprismatic substance; and the angle of extinction will now range from 22·5 to 40°.

The suggested arrangement of the crystallites in the interprismatic substance will also account for the brightness in transverse section; for consideration of figure 5, which is equally well applicable to the interprismatic substance as to the prism material, will show that when the second group of crystallites is predominant, the maximum intensity transmitted reaches an appreciable percentage of the standard intensity, and the interprismatic substance will therefore not appear dark except in the extinction position.

The structure proposed for the interprismatic substance is illustrated, side by side with that of the prism material, in figure 6. The short lines represent the directions of the crystallite axes and not their positions, the latter being unknown.

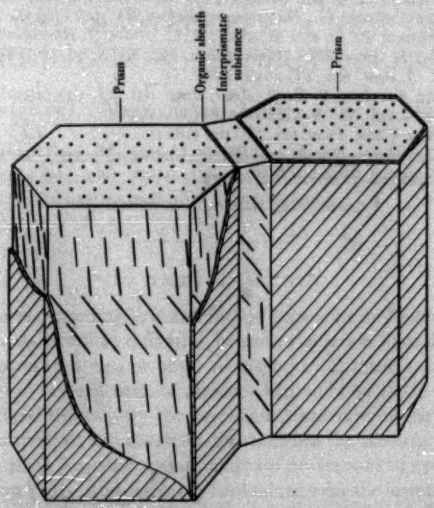

Figure 6. Diagram illustrating proposed structure of prism material and interprismatic substance. The directions of the crystallite axes are represented by short lines.

§ 5. ACKNOWLEDGEMENTS

The work forms part of an investigation being carried out on behalf of the Dental Disease Committee of the Medical Research Council, to whom the author is indebted for permission to publish it. He is also indebted to Dr G. W. C. Kaye, F.R.S., for the interest which he has taken in it.

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THE THERMAL MANOMETER, A NEW DEVICE FOR RECORDING LOW ABSOLUTE PRESSURES

By R. S. VINCENT, M.Sc., A.M.I.E.E. and A. SIMONS, M.Sc.

ABSTRACT. An instrument for recording absolute pressures over the range from 0·1 to 100 mm. of mercury is described. The boiling point of mercury, boiling under the pressure to be measured, is recorded by means of a thermojunction.

§ 1. INTRODUCTION

The problem of recording low absolute pressures by any direct method is one of considerable practical difficulty, because the power available is usually insufficient to move a pen or other recording mechanism satisfactorily in the pressure range up to say 100 mm. of mercury. In addition to the above difficulty, instruments operated directly by the mechanical pressure of the rarefied gas are characterized by a more or less linear relation between the absolute pressure and the movement of the pen. For this reason, and also on account of zero error and pen friction, a very poor percentage accuracy is obtained near the lower end of the scale. For example, when a range up to 100 mm. of mercury is covered, readings of the order of 10 mm. may easily be in error by 10 per cent owing to pen friction and mechanical imperfection of the mechanism, while readings of the order of 1 mm. would be almost meaningless, and it would be impossible to detect an absolute pressure, or pressure-change, of 0·1 mm.

To meet the need for an accurate and reliable method of recording absolute pressure in gaseous media, the writers have developed the thermal manometer.

§ 2. EARLY EXPERIMENTS

The first conception of the principle of the thermal manometer was to balance the absolute pressure to be measured by the saturated pressure of a liquid, and to record the temperature of the saturated vapour necessary to produce this balance. A mercury-in-glass U-tube manometer was arranged as in figure 1. The closed limb was provided with an external heating coil through which a current passed when the circuit through the contact in the right-hand limb was broken. The effect of the heating current was to raise the vapour pressure of the mercury in the left-hand limb until it became equal to the absolute pressure applied to the right-hand limb. An approximate state of balance was thus maintained. By recording the temperature of a thermojunction in the left-hand limb, a record of the applied absolute pressure was obtained. It was found to be difficult to avoid hunting in the above arrangement, so that the temperature record showed a regular periodic fluctuation even when a steady absolute pressure was applied to the manometer.

To overcome the above difficulties, it was decided to supply heat steadily to the liquid and to disperse the excess heat in a vertical condensing tube. The mercury U tube, the only purpose of which is to operate the contact, thus became unnecessary, and the very simple form of apparatus described below was evolved.

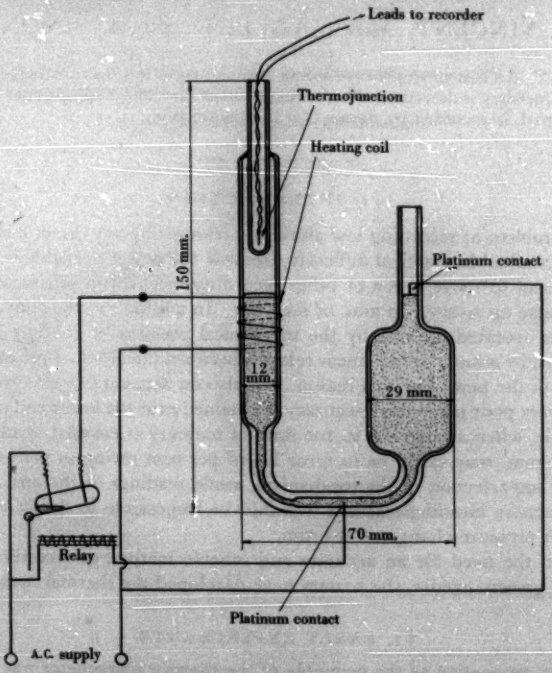

Figure 1.

§ 3. THERMAL MANOMETER WITHOUT CONTACTS

Description of the manometer. The instrument is shown in figure 2, it consists of a tall glass tube about 1 in. in diameter, closed at the bottom and having a side tube at the top for connecting to the pressure to be measured. A narrow tube, also sealed at the bottom, runs down the centre of the wide tube to within about 3 in. of the bottom. A small quantity of mercury is kept boiling by an external heater in the bottom of the larger tube. The vapour condenses on the sides of the tube and returns to the boiler. The condensation temperature of the vapour is recorded by a thermocouple passed down to the lower end of the inner tube, a Cambridge thread recorder reading from 0 to 300° C. being used for this purpose.

It is clear from the diagram that the temperature of the saturated vapour over the boiling liquid will be maintained automatically at a value such that the saturated vapour pressure is equal to the absolute pressure applied. Any slight excess of

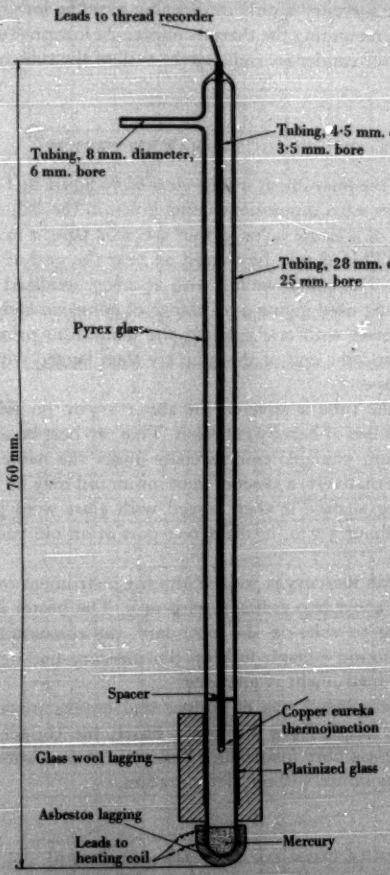

Figure 2. Diagram of thermal manometer.

temperature would cause an increase in the flow of vapour into the condensing tube, and therefore an increase in the rate of losing heat and a return to the equilibrium temperature. Similarly, a temperature slightly below the equilibrium value would cause a reduction of the flow of vapour into the condenser, with a corresponding

reduction in heat-loss and consequent rise of temperature up to the equilibrium value. The supply of heat to the mercury is such that the mercury is just boiling at the highest pressure which it is desired to measure—in this case about 100 mm. In these circumstances, the mercury is only condensing over the lower end of the tube just above the lagging surrounding the thermocouple. At low pressures the mercury boils faster and the vapour condenses right up the wall of the tube nearly to the top.

§ 4. CONSTRUCTIONAL DETAILS

The construction of the instrument will be clear from figure 2. The body is made of Pyrex glass, all the essential dimensions being given in the diagram. The heater is wound on a bedding of a single layer of thin asbestos tape, 1 in. wide, to give a firm grip on the glass. This should be wound as near the end of the tube as the curved surface permits. The coil is wound with 26 s.w.g. oxidized nichrome wire, three metres of wire being used giving a resistance of approximately 20 ohms. The turns are wound in contact with one another, the oxide film providing sufficient insulation. The heater and the end of the tube are then lagged with a layer about 1 in. thick of asbestos tape.

The outer wall of the tube is silvered for about 1·5 in. on either side of the thermocouple to prevent loss of heat by radiation. This can best be done by platinizing, the colloidal platinum available commercially under the name "liquid bright silver" being used. Alternatively, a sheet of aluminium foil may be wrapped round the glass. The reflecting surface is then lagged with glass wool packed inside a corrugated metal tube about 3·5 in. in diameter; part of an old vacuum-flask case serves very well.

Finally, sufficient clean mercury is poured into the instrument to fill it up to the top of the heating-coil; about 200 grams is necessary. The heater is supplied by a small transformer giving 30 volts on the secondary, the consumption thus being 45 watts. This will be found enough to keep the mercury boiling at the highest pressures for which the instrument is intended.

The thermocouple employed was of copper and eureka metal. In order to insulate the two thermocouple leads from each other, the copper wire only was provided with braided glass fibre insulation. This type of insulation was found to withstand the high temperature without deterioration.

§ 5. OPERATION OF THE MANOMETER

A calibration curve showing the relation between temperature and pressure is shown in figure 3. In obtaining this curve the pressures up to 10 mm. of mercury were measured with a McLeod gauge, and from 10 to 100 mm. with an ordinary U gauge. It will be seen that between 1 and 100 mm. of mercury the curve follows the known mercury-vapour {pressure, temperature} relation very closely.

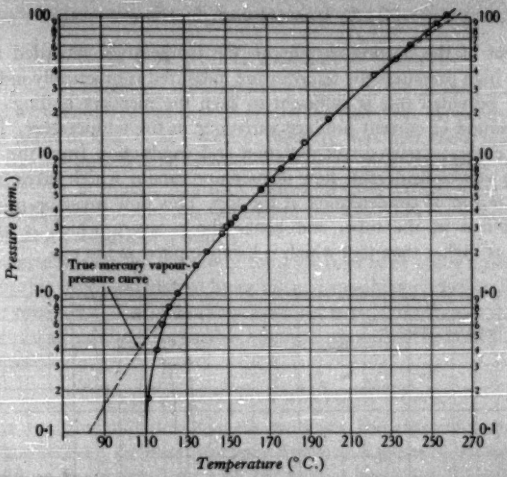

Figure 3. Calibration curve of thermal manometer. Heating current 1·5 amp.

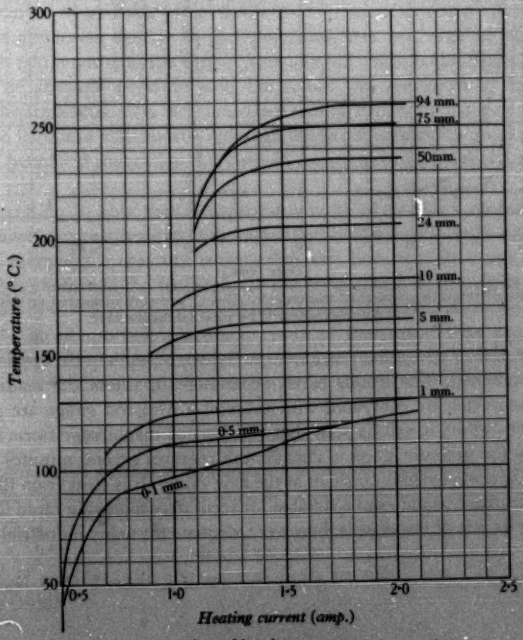

Figure 4. The effect of heating current on temperature.

The effect of the heating-current on the temperature recorded is shown in figure 4. At high pressures the temperature measured is practically independent of the loading, provided this is sufficient to keep the mercury boiling. But at low pressures changes of current produce variations in the temperature. This may be due to two causes; firstly direct radiation and conduction from the heater, and secondly the very considerable flow of mercury vapour which takes place up the tube to the region of condensation at the top. This flow must be produced by a pressure-gradient in the tube, and hence the pressure which is affecting the temperature is higher than that which is being applied at the top.

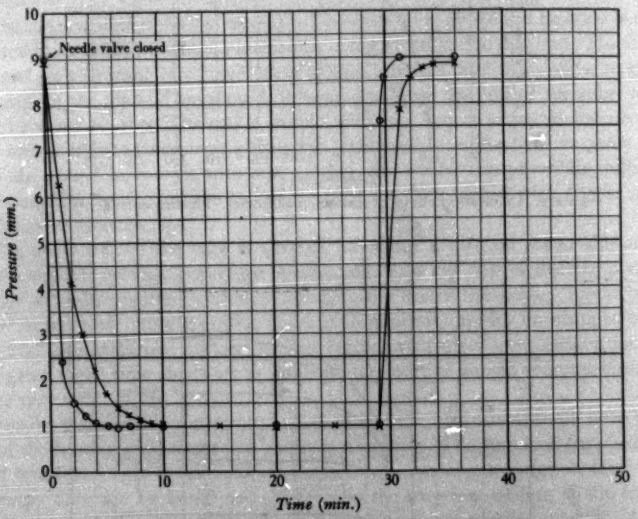

Figure 5. Rate of response of thermal manometer. ○—○ pressure measured on McLeod gauge. ×—× pressure measured by thermal manometer.

The rate of response of the instrument to sudden changes in the applied pressure has been examined. The results of an experiment in which the readings of the thermal manometer are compared with those of a McLeod gauge are graphed in figure 5, from which it will be seen that the thermal manometer starts to respond immediately the pressure starts to change, but requires several minutes before the reading has become sensibly constant at the new value. It is clear from these results that the instrument would not be suitable for accurately following rapid fluctuations of pressure; its rate of response is, however, adequate for many industrial processes.

§ 6. THE USE OF ALTERNATIVE LIQUIDS

This difficulty of obtaining readings independent of the heating-current at low pressures led to attempts to use liquids other than mercury. Apieson oil A, which has a boiling point of 192° C. at 1 mm. of mercury, was first tried, but exactly the same effect was produced. The change of pressure over the range from 0·01 to 0·1 mm. of mercury produced very little change in temperature. This was the case even when the heating-current was reduced to the minimum necessary to keep the liquid boiling at a pressure of 1 mm.

A flat heater covering the bottom of the tube instead of being around the sides was tried, but this produced violent bumping and splashing of the liquid, especially at the higher pressures. Since the range from 0·1 mm. upwards was that for which the instrument was required, no further attempts were made to extend the range to lower pressures.

§ 7. REFERENCE TO PREVIOUS WORK

Since the experimental development work on the thermal manometer was carried out, the writers' attention has been drawn to a paper by Hickman, Hecker and Embree[1] on the measurement of low vapour pressures. This describes the measurement of the vapour pressures of diffusion-pump liquids such as butyl phthalate over the range from 5 mm. down to 10^{-4} mm. of mercury. For the higher pressures an apparatus similar to that described here was used to measure the condensation temperature of the vapour from a liquid which boiled under controlled pressure. The absolute pressure over the liquid was measured by instruments external to the apparatus. It is mentioned that such an apparatus responds sufficiently quickly to be used as a manometer, although it was not actually so used in the work described. There is no suggestion that such an apparatus could be used for recording purposes.

It is interesting to note that the apparatus in its simple form was found to be unsuitable for the measurement of condensation temperatures at pressures below 0·05 mm. of mercury for the same reasons that have been described here. Our own experience has confirmed the tendency for the temperature to become independent of the pressure at very low pressures, and it thus appears that the thermal manometer is unlikely to be of use below about 0·1 mm. of mercury. There are, however, important industrial applications requiring absolute pressure to be recorded in the range from 0·1 to 100 mm., and for this range the thermal manometer is eminently suitable.

§ 8. ACKNOWLEDGEMENT

The authors wish to thank Callenders Cable and Construction Co., Ltd., in whose laboratories the experimental work was carried out, for permission to publish this paper.

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THE ELECTRIFICATION OF POWDERS BY FREE FALL

By G. MORRIS, B.Sc.

Received 31 *May* 1939

ABSTRACT. The electrification of insulating powders due to free fall has been examined quantitatively. For a given powder the quantity of electricity generated is found to be proportional to the square root of the product of the height of fall and the weight of powder. It is independent of the nature of the surface of the receiving vessel, and of the capacitance of the measuring system. Under given conditions of fall the quantity of electricity generated increases rapidly as the fineness of the powder is increased.

§ 1. INTRODUCTION

In a recent paper[1] Sir Ambrose Fleming has described the generation of electrostatic charges by the fall of insulating powders. Some ten years ago the author made some quantitative measurements of this effect, and obtained results which throw some light on the mechanism of generation of such charges.

§ 2. DESCRIPTION OF THE APPARATUS

The apparatus consisted of an earthed brass tube, 1 in. in diameter and 12 in. long, mounted axially above a Faraday vessel consisting of two cylindrical lead vessels, the outer of which was 9 in. in diameter and 12 in. deep and was earthed, while the inner was 7 in. in diameter and 9 in. deep, the vessels being connected to the measuring system, which usually consisted of a Dolezalek electrometer and a standard mica condenser, figure 1.

To the top of the brass tube was fixed a cone of cotton of 11 cm. slant side with a ring of copper wire, 10 cm. in diameter, sewn into the upper edge. This ring could be raised or lowered from a distance by means of a system of insulated strings.

A known weight of powder was placed in the annular bag formed by the lowered cone, the measuring system was earthed and insulated, and when the cone was stretched, the powder fell down the tube into the receiver. In the majority of the experiments a large nickel beaker was placed in the inner lead vessel to catch the falling powder. When fine powders were being handled it was found to be necessary to stretch the cone with a sharp jerk to loosen the small quantity of powder which adhered to the cloth cone.

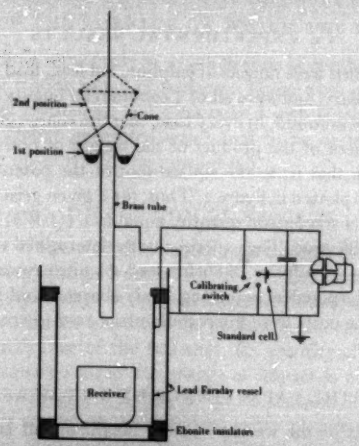

Figure 1. Apparatus for the measurement of the mechanical electrification of powders.

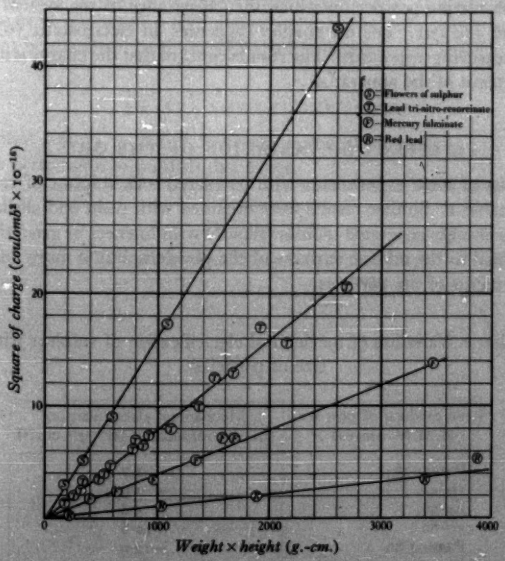

Figure 2. Relation between square of charge and weight × height.

§ 3. EXPERIMENTAL RESULTS

The powders studied were flowers of sulphur, red lead, lead tri-nitro-resorcinate, and fulminate of mercury, and were all of a commercial quality and grist. They were dried before use. It was found, in every case, that the charge generated was proportional to the square root of the product of the weight of powder and the distance through which it fell, that is, to the square root of the potential energy lost. The experimental data are plotted in figure 2. Thus, for a given grist of powder of a given substance, there is a characteristic quantity equal to $\sqrt{(Q/Wh)}$ which may be called the mechanical electrification. The experimentally determined values of this quantity were 12·6 for flowers of sulphur, 8·86 for lead tri-nitro-resorcinate, and 6·36 for mercury fulminate, the powders being negatively charged, and 3·36 for red lead with a positive charge. The units were microcoulombs $\times 10^{-4}$ $(\text{g.cm.})^{-\frac{1}{2}}$.

§ 4. EFFECT OF THE CAPACITANCE OF THE MEASURING SYSTEM

Since the product of the weight and the height of fall is proportional to the mechanical energy of fall, and the square of the charge generated is proportional to the electrical energy of the system, experiments were made, flowers of sulphur being used as the insulating powder, in which the capacitance of the system was varied from 10μF. to 180$\mu\mu$F. For the smaller capacities an electrostatic voltmeter was used as the measuring instrument. The results confirm that the charge is independent of the capacitance of the apparatus.

§ 5. EFFECT OF VARIATION OF THE GRIST

The results of some experiments with powdered rock sulphur of various grists are given in table 1. Similar results were obtained with resin and shellac.

While the charges generated increase rapidly with fineness of grist, the charge is not proportional to the relative surfaces of the powders, but is more nearly related to the mean radius of the particles.

Table 1. Variation of mechanical electrification of sulphur with size of particles

Sieve grading of powder (I.M.M. sieves)	Mechanical electrification (microcoulombs $\times 10^{-4}$ $\text{g}^{-\frac{1}{2}}$ $\text{cm}^{-\frac{1}{2}}$)
Retain 16	− 2·7
16–24	− 3·9
24–50	− 5·8
50–80	− 10·6
Passing 80	− 12·0
Flowers of sulphur	− 12·6

§ 6. EFFECT OF THE SURFACE ON WHICH THE POWDER FALLS

Some determinations were made in which flowers of sulphur was allowed to fall into receivers having a wide range of surfaces. These were polished nickel, glass, enamelled iron, rough cardboard, and glass coated with sulphur or bitumen.

The quantity of charge generated was found to be quite independent of the nature of the surface on to which the powder fell.

§ 7. DISCUSSION OF RESULTS

The linear graphs of figure 2, all of which pass through the origin, demonstrate that neither the tipping system from which the powder is released nor the brass tube through which it falls plays any part in the generation of the charge, so that the charge must be caused by the impact of the powder at the end of the fall.

The physical significance of the fact that the quantity of charge generated is proportional to the square root of the energy of impact is not apparent. An explanation would probably give a picture of the mechanism of charge-generation.

That the charge is apparently independent of the surface on which it strikes is probably explained by the fact that only a small proportion of the powder actually strikes the receiving surface, the greater proportion of the powder grains landing on grains of the same powder which had arrived previously. It would be of interest to repeat the experiment of § 6 with decreasing quantities of powder, when different results might be observed as the weights falling became very small.

§ 8. ACKNOWLEDGEMENTS

The author wishes to thank Imperial Chemical Industries, Ltd. (Explosives Group), for permission to publish this paper.

REFERENCE

(1) FLEMING. *Proc. Phys. Soc.* **51**, 402 (1939).

THE INTERMEDIATE IONS OF THE ATMOSPHERE

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Received 22 *March* 1939. *Read in title* 23 *June* 1939

ABSTRACT. Evidence is produced to show that (*a*) the smaller nucleated ions in city air exist in separate groups with distinct mobilities; (*b*) the variation of the mobility of these ions with humidity is such as to lead to the conclusion that the ions are composed of droplets of sulphuric acid; (*c*) it appears that the ions result from the aggregation of particles containing about 2200 molecules of sulphuric acid or multiples thereof. The acid, before and after aggregation, is hydrated to an extent dependent on the temperature and the humidity of the atmosphere and on the radius of the droplet.

§ 1. INTRODUCTION

The ions of the lower atmosphere are broadly grouped into two classes generally referred to as "small" ions and "large" ions. The small ions have approximately unit mobility (1 cm.sec.$^{-1}$/v. cm.$^{-1}$). They appear to be composed of clusters of a few gas molecules, and have only a narrow range of mobilities. The term "large ions" is applied in its widest sense to a group of atmospheric ions ranging in mobility from 1/50 to 1/10,000 or less. These ions almost certainly consist of liquid or solid nuclei which have become charged usually by the attachment of a small ion. Whilst the terms "small ions" and "large ions" are in common use, it is perhaps desirable to speak of "gas" ions and "nucleated" ions in order to avoid confusion when it becomes necessary to divide the nucleated ions into size groups.

As a result of experiments by Langevin[6], nucleated ions were first considered to be a homogeneous species with a mobility of about 1/3000. Later Pollock[8] concluded that the nucleated ions were made up of two distinct and non-overlapping species, viz. the so-called intermediate ion of mobility 1/50, and the previously mentioned Langevin ion. R. K. Boylan[1] then brought forward evidence for the existence of ions down to a mobility of 1/15,000. The work of Israel and Schulz[4] led to the conclusion that nucleated ions form a continuously graded series of sizes varying in mobility from 1/100 to 1/15,000. The present paper has for its object the examination of the mobility of the faster ions in this range, often termed intermediate ions, by means of apparatus of improved resolving power.

§ 2. METHOD

Methods for the measurement of the mobility of the natural atmospheric ion are limited to those of the blast type, because of considerations regarding the supply of ions. All blast types have considerable disadvantages, but the divided electrode method due to Zeleny[14] is amongst the more satisfactory of its kind. This apparatus consists of a cylindrical condenser, the inner electrode of which is divided into two unequal portions insulated from each other. The longer portion A, figure 1, is connected to the earth; the shorter portion B is connected to a sensitive electrometer. A current of air of constant known velocity ϕ cm.^{3}/sec. is drawn along the tube in the direction of the arrows, figure 1; successively increasing potentials V are applied to the outer cylinder, and the resultant currents i flowing to the short electrode are measured. With the single species of ions of mobility W and concentration N the theoretical graph of $\{i, V\}$ is of the type shown in figure 2*a*, i.e. the

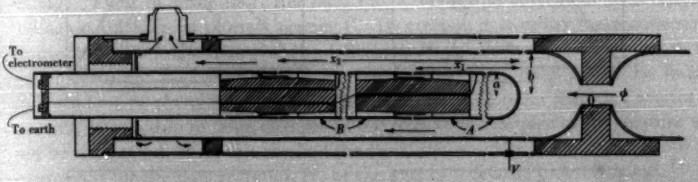

Figure 1. Diagram of the divided condenser. The electrode is shown sectioned in two places.

current at first increases linearly along OA to a maximum (at potential V_1), and then decreases also linearly along AB to zero (at potential V_2). The lines OA and OB are represented respectively by the equations

$$i = 4\pi NWeVC_2 \qquad \text{.....(1)}$$

and

$$i = 4\pi NWe\,[V_1(C_1+C_2) - VC_1], \qquad \text{.....(2)}$$

where C_1 is the inter-electrode capacity of the front section of the condenser, C_2 is the inter-electrode capacity of the rear section and e the charge carried by an ion. In electrostatic measure $C_1 = \frac{1}{2}x_1/\log(b/a)$ and $C_2 = \frac{1}{2}(x_2 - x_1)/\log(b/a)$, where x_1 is the length of the front section of the inner electrode, $x_2 - x_1$ is that of the rear section, a is the external radius of the inner electrode and b is the internal radius of the outer.

The mobility of the ions is given by either of the expressions

$$W = \phi/4\pi V_1(C_1+C_2) \qquad \text{.....(3)}$$

and

$$W = \phi/4\pi V_2 C_1, \qquad \text{.....(4)}$$

ϕ being the rate of air-flow in cm.^{3}/sec.

From equations (3) and (4) can be obtained an important but generally overlooked criterion of the reality of the observations, viz.

$$V_2/V_1 = (C_1+C_2)/C_1. \qquad \text{.....(5)}$$

The concentration of the ions is derived from (1) and (3) as

$$N = E_m R\,(C_1 + C_2)/\phi e t, \qquad \ldots\ldots(6)$$

where E_m is the increase of potential of the electrometer system in time t for the applied voltage V_1, $R = (C_2 + C_3)/C_2$, and C_3 is the capacity of electrometer and its connexions.

In dealing with mixtures of ions the curves due to the individual species will be additively combined. The exact shape will depend, *inter alia*, on the relative numbers

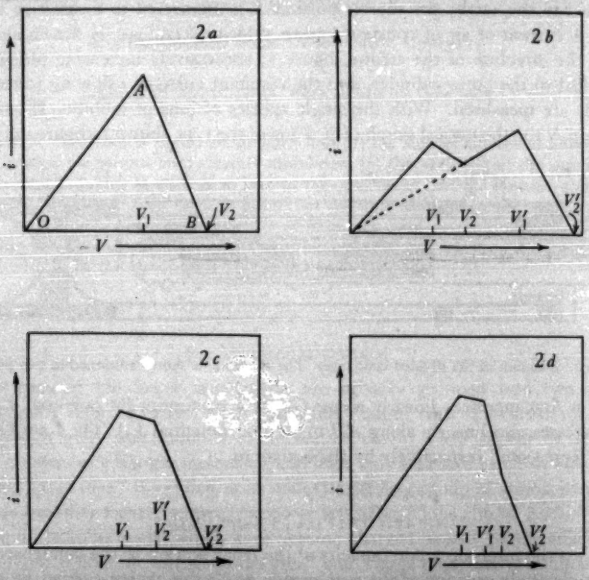

Figure 2. Types of curves theoretically obtainable with the apparatus, with (*a*) single homogeneous species, and two species of mobility W and W', where (*b*) $W/W' > (C_1 + C_2)/C_1$, (*c*) $W/W' = (C_1 + C_2)/C_1$, and (*d*) $W/W' < (C_1 + C_2)/C_1$.

and mobilities of the various ions present. Whether or not it is possible to detect a given type of ion in a mixture depends largely upon its relative concentration. Thus, in the present case, unless the ion type represents at least 2 per cent of the total ions present it will probably escape detection. Another factor is the dimensions of the apparatus, and to illustrate this, mixtures of two kinds of ions of mobility W and W' may be considered. Individually either ion will give rise to curves of the type shown in figure 2*a*. When mixed, the ions will give curves of the types shown in figures 2*b*, 2*c*, or 2*d*, dependent on the relations between the mobilities of the ions and the capacities of the apparatus. Whilst the critical potentials may be found from figures of types 2*c* and 2*d*, where $W/W' \leqslant (C_1 + C_2)/C_1$,

they may be located more readily with type 2*b*, where $W/W' > (C_1+C_2)/C_1$. The ratio $(C_1+C_2)/C_1$ thus defines a practical limit to the resolving power of the apparatus. Thus, unless adjoining ion groups have their mobilities W and W' such that $W/W' > (C_1+C_2)/C_1$, their resolution is unlikely to be accomplished if they are mixed with relatively large numbers of other ions. If only two ion groups were involved, then W and W' might be obtained from V_1 and V_2', but unless V_2 and V_1' could be definitely located it would be uncertain whether the ions dealt with were individual species, or members of a series forming a band of mobilities. Theoretically, resolution is improved by making $(C_1+C_2)/C_1$ approach unity, but this is limited by the practical necessity of having C_2/C_3 sufficiently great to allow of the number of ions collected by the second electrode being readily measured.

The foregoing discussion assumes that N remains constant throughout the series of observations. This assumption rarely holds in practice. To allow for this, simultaneous observations may be made with a second condenser held at constant voltage. Then in place of i, the ratio i/i_c, i_c being the current in the constant-voltage condenser, is plotted against V. In practice it is rather more convenient to use not i/i_c but the ratio p of the deflections of the electrometers attached to the variable-voltage and constant-voltage condensers. This ratio p will be largely independent of changes in the ion content, although if more than one species be present, effects due to large changes in the relative proportions of the mixture will not be eliminated. With this method the mobilities are calculated as before, but the expression for the concentration N_g of the ions of a definite group becomes

$$N_g = p_m \bar{E} s R\,(C_1+C_2)/(\phi e t),$$

where p_m is the maximum change in p due to ions of a particular group, $\bar{E}$ is the average deflection of the electrometer attached to the constant-voltage apparatus, and s is the sensitivity (E.S.U. per division) of the electrometer on the variable-voltage apparatus.

§ 3. APPARATUS

The divided electrode employed is shown diagrammatically in figure 1. To prevent the turning back of the ions by the field, the inlet orifice O was made of such a size that the velocity of the entering air current was greater than any velocity imparted to the ions by the field. A Wulf unifilar electrometer was employed to measure the currents whilst the potentials applied to the condenser were checked by an electrostatic voltmeter. The sensitivity of the electrometer, usually 200 divisions per volt, was frequently checked by a voltmeter during a series of observations. Apart from containing only one large central electrode, the control condenser was similar to the divided-electrode apparatus. Arrangements were made to photograph the indications of the control apparatus so that, when desired, a record could be obtained of the concentrations of the ions. The air, before reaching the control electrode, was deprived of its small ion content by passage through a fore-condenser charged to a low voltage. The air was drawn into the two condensers through a short length of brass tubing, 5 cm. in diameter, capped with gauze of

1 mm. mesh, and projecting through the wall of a wooden hut, at a point about 2 m. above the ground level. The air velocity was measured by a plate-orifice arrangement. A vacuum cleaner fan, working at a speed somewhat below the rated value, served to draw air uniformly through the apparatus. Fine control was obtained by valves in the air lines, and no serious difficulty was encountered in maintaining a measurably constant air flow. In any case, the effect of minor fluctuations in the blast would be taken account of, up to a point, by the control condenser, as the fan provided suction for both condensers.

In order to determine one point on a curve, ions were collected, in both condensers, for a period ranging from 30 to 180 sec., according to circumstances. The range of mobility investigated varied considerably from experiment to experiment with the ion concentration. Thus on those occasions when the ion content was so small as to require 180 sec. to give a satisfactory electrometer reading, the range investigated had to be curtailed. The determination of a complete curve generally required from 1½ to 2 hr.

§4. RESULTS

The measurements, which were made at Kew Observatory, were carried out during the winter of 1937–8, and were confined to the observation of positive ions. The curves obtained were generally of the type shown in figure 3, which represents an actual set of observations made. This curve exhibits several small, but apparently definite discontinuities, and from considerations of the previous section, has the appearance of being a composite curve arising from the presence of various more or less distinct groups of ions in the mobility range 100 to 10×10^{-4} cm. sec.$^{-1}$/v. cm.$^{-1}$ A total of 123 discontinuities was observed in 48 satisfactory sets of observations. The measured mobilities did not fall into a few well-defined groups, but were scattered over the whole range investigated. When observations were repeated at intervals of a few hours, a satisfactory agreement was obtained; for instance, on 28 February 1938, the ions of the following mobilities were detected:

At 15 hr.	0·0120	0·0045	0·0026	0·0016
At 17 hr.	0·0110	0·0049	0·0027	no observation

and on February 7, with different air flows

At 11 hr. ($\phi = 333$)	—	0·0067	0·0041	0·0026
At 12 hr. ($\phi = 500$)	0·0145	0·0074	0·0039	0·0021

In this second example the more favourable working conditions associated with the greater air-flow resulted in the detection of a faster ion. Generally speaking, there was no difficulty in obtaining regular curves, but on twelve occasions out of a total of sixty, the curves obtained showed irregularities. Of these twelve occasions, two were associated with instrumental defects, five occurred when observations were attempted in showery weather, and three during fog or mist, whilst two could not be accounted for. Table 1 shows the 48 sets of satisfactory observations of mobility together with the associated values of the ion-concentrations.

Table 1. Observed mobilities W (10^{-4} cm.sec.$^{-1}$/v. cm.$^{-1}$), and concentrations N_g (ions/cm.^{3}) of the ion groups

Date 1937	G.m.t. (hr.)	W	N_g	Date 1938	G.m.t. (hr.)	W	N_g
October 26	16 –17	70	580	January 3	11 –12	39	300
28	10½–12½	100	340			23	670
29	10½–11½	35	640			15	750
		21	900		14 –17	135	200
29	13 –18	36	600			74	240
		18	900			35	390
29	20½–22	100	700			23	600
November 2	10½–11½	36	600			15	850
3	12 –13	71	160			10	1100
5	10½–11½	77	180	5	10½–12	55	200
6	00 –01½	91	240			34	370
10	21 –23	88	280			18	1000
11	9½–10½	85	340			12	—
15	10½–11½	82	560	31	14½–16	72	170
16	10½–12	82	1000			34	170
		43	1200			20	170
		28	1600	February 2	10 –11	38	330
17	11 –12½	38	550			18	680
18	14½–17	70	150	7	10½–11½	67	250
		49	200			41	300
		31	380			26	360
		20	770			19	800
22	14½–16½	77	290	7	11½–12½	145	210
		52	360			74	360
		34	430			39	590
25	11½–12½	48	520			21	—
		37	630	8	10½–11½	61	270
		27	—			25	360
26	10 –12	23 (*a*)	1300	11	15½–16½	61	240
		49	1100			25	360
		23 (*b*)	1350			17	1050
26	15 –16	41	730	12	16½–17½	57	370
		17	600	22	10 –11	160	75
27	10 –11	26	1730			86	150
		13	—	22	11½–13	140	190
December 6	10½–11½	61	390			56	380
		25	1050			26	380
		13	1100	28	11½–13	130	140
7	10½–13	51	580			45	490
		36	720			23	430
		26	970	28	14½–16	120	200
		16	1350			45	300
		10	1550			26	330
9	15 –17	220	240			16	—
		94	350	28	16½–17½	110	180
		55	—			49	400
11	21 –22	64	170			27	430
13	10½–12½	78	210	March 5	10 –11½	28	200
		38	200			17	520
14	15 –16½	54	400			9	—
		31	440	12	11 –13	50	200
15	10½–13	43	230			35	390
		33	500			24	580
		20	500			13	900
		13	800	20	10 –11½	84	230
22	11½–13	65	950			45	230
		39	1400			27	450
		35	2500			16	530
29	10½–13	220	180			12	740
		120	270				
		71	190				
		39	440				
		26	450				
30	10 –11	126	—				

§ 5. DISCUSSION OF OBSERVATIONS

(*a*) *Relations between mobility and humidity.* The main evidence in favour of regarding the observed discontinuities in the $\{p, V\}$ curves as being due to individual ion species, comes from a consideration of the value of V_2/V_1. Theory requires this to be equal to $(C_1+C_2)/C_1$, equation (5). The results showed that V_2/V_1 averaged 1·32, independently of V_1, whilst $(C_1+C_2)/C_1$ was 1·26. Processes such as diffusion would tend to make V_2/V_1 greater than the theoretical value. The fact that the mobility results were repeatable, even with different air-flows, also indicates that the discontinuities were not the result of chance.

The ions examined in this investigation represent a fraction of the Aitken nuclei of condensation. The Aitken nuclei are generally regarded as being droplets of solutions of hygroscopic substances suspended in the atmosphere. A droplet of hygroscopic solution, when placed in humid air, will tend to a condition where its partial

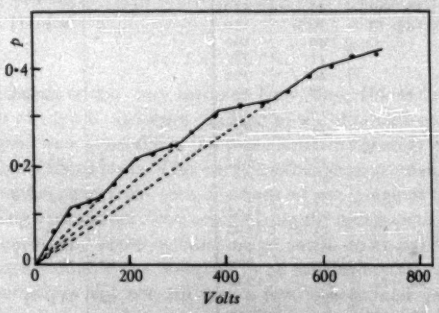

Figure 3. Typical experimental curve.

pressure of water vapour will be equal to the partial pressure of the water vapour in the air. This concept has been applied by Köhler[5], developed by Wright[12], and finds some experimental support from Scrase's observations[9] of the relations between charged and uncharged nuclei and the atmospheric humidity. It follows that, at constant temperature, a droplet containing a given quantity of solute can assume a whole series of sizes, dependent, *inter alia*, upon the partial pressure of water vapour in the air. If such a droplet becomes charged (as for example by the absorption of a gas ion from the air), it will give rise to an ion the mobility of which will vary continuously with the humidity. From the mobility it is possible to calculate the radius of the ion; then if the composition of the solute, together with various physical properties of the solution, and the temperature and humidity of the air be known, the quantity of the solute in the droplet may be obtained.

Whilst the composition of the solute was not known for certain, various considerations made it appear likely that the ions in this investigation were composed principally of sulphuric acid droplets. Thus, the total concentration of the ions N

varies diurnally in a manner which can be largely accounted for in terms of atmospheric turbulence, and the extent of domestic combustion processes (see table 2). Sulphuric acid is amongst the products of combustion of such sulphur-containing fuels as are frequently used in domestic activities; moreover Wright[12], using unpublished data by Coste and Courtier, indicates that sulphuric acid is present in London air to an extent which would account for all the nuclei present. Although the spraying of sea water can give rise to nuclei, it is unlikely that any notable proportion of the nuclei at Kew is of marine origin. Thus Parkinson[7] gives the total nucleus-content of ocean air as a few hundred per cm^3, whilst at Kew the nuclei amount to an average of 30,000 per cm^3. Wright and Coste[13] have indicated oxides of nitrogen in aqueous solution as a possible constituent of nuclei, but as the physical properties of sulphuric acid are much better known than those of unstable nitrous solutions, it was thought desirable initially to treat the nuclei as being composed of sulphuric acid.

Table 2. Diurnal variation of total positive ions at Kew

G.m.t. (hr.)	01	03	05	07	09	11	13	15	17	19	21	23
N ($cm.^{-3}$)	10·1	8·4	8·6	9·7	10·2	7·5	8·0	8·4	9·6	14·5	16·2	$13{\cdot}9 \times 10^3$

This table represents the means of seven complete days of continuous observations made in April and May 1938.

In the present investigation the radius r of an ion was obtained from its mobility W by the use of Thomson's expression in the numerical form

$$W = (1 + 10^{-5}/r)/(2{\cdot}15 \times 10^9\, r),$$

whilst the quantity of solute in the droplet was derived by a method due to Wright. In preference to employing the generalized equations given by Wright[12], use was made of the actual vapour pressures, densities, and surface tensions of sulphuric-acid solutions as set out in the *International Critical Tables*. The calculations required a knowledge of the charge on the ions. This has been shown to be equal to the charge on the electron[2] for ions in this range of mobilities. On the basis of the above considerations isothermals were drawn showing the relation between humidity and mobility for ions containing various quantities of sulphuric acid. The quantity of acid has been expressed in terms of r_0, the radius of the droplet of pure sulphuric acid which can be considered as having given rise to the nucleus by the absorption of water vapour. An example of one set of such isothermal curves is shown in figure 4. From these isothermals, and the knowledge of the vapour pressure and temperature obtaining during the measurements, it was possible to draw a curve for each observation period showing, for the particular hygrometric state of the atmosphere obtaining at the time, the relationship between r_0 and mobility. These curves were then used to obtain values of r_0 from the measured mobilities.

Values of r_0 were obtained for the mobilities W of the ions detected on each occasion, and a graph of W against r_0 was constructed, figure 5. Figure 5 shows

that, whilst the values of W are fairly evenly distributed, there are notable gaps in the distribution of r_0. Various values of W are associated with a relatively few particular values of r_0, which are denoted by horizontal lines. This is especially noticeable with the smaller values of r_0; as higher values of r_0 are reached, they fall so closely together that it is difficult to deduce any preferred values. For this reason twenty observations for which $r_0 > 10^{-6}$ have been excluded from the diagram. There have been omitted also two observations which both gave $W = 220 \times 10^{-4}$ and $r_0 = 2{\cdot}9 \times 10^{-7}$, and which were regarded as being insufficient to establish another preferred value for r_0. Apart from the above-mentioned exceptions, figure 5 contains all the satisfactory observations of W and r_0. From

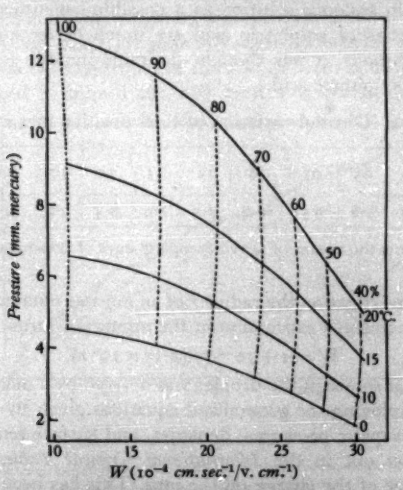

Figure 4. Isothermal curves for a sulphuric acid ion for which $r_0 = 9{\cdot}5 \times 10^{-7}$ cm. showing the variation of mobility with humidity.

figure 5 it may be seen that, as a whole, the ions investigated show no tendency to fall into groups on a mobility basis. On the other hand, the figures for r_0 show a definite tendency to be represented by relatively few values.

In figure 6 the cubes of the preferred values of r_0 are plotted against the series of numbers $g_1^n = 1, 2, 3, \ldots, n$. A straight line may be drawn through the points. Thus it would appear that the r_0 groups observed have arisen, directly or indirectly, from the agglomeration of primary particles with values of r_0 equal to $3{\cdot}6 \times 10^{-7}$ cm., corresponding to about 2200 molecules of sulphuric acid. The value of g may be regarded as an aggregation factor, giving the number of primary particles going to make up the ion with which it is associated. This value will sometimes be used in the sequel to designate a particular kind of ion more or less irrespective of its mobility. There is perhaps some doubt as to whether the particle with r_0 equal to

$3{\cdot}6 \times 10^{-7}$ cm. is actually the unit, or whether it might not be built up from two $2{\cdot}9 \times 10^{-7}$ cm. particles; the latter is the size mentioned as having been excluded from the diagram, and $(3{\cdot}6/2{\cdot}9)^3 = 1{\cdot}92$; but the remaining particles apparently differ by steps corresponding to the volume of the $3{\cdot}6 \times 10^{-7}$ particle. There is available no exact numerical information as to the dynamics of the combination

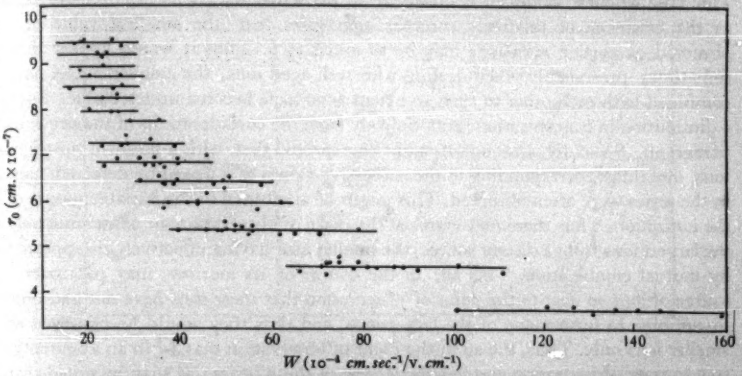

Figure 5. Curves showing the relation between ion-mobility and the radius of the droplet of pure sulphuric acid from which the ion may be considered as having been derived.

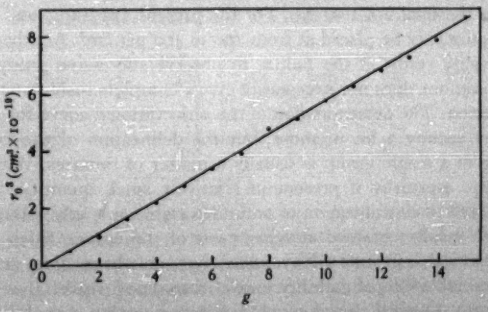

Figure 6. Curve illustrating the growth of ions by agglomeration.

of the particles, but certain approximate calculations show that such combination could occur at a rate sufficiently large to account for the measured ionic concentrations. Thus, if it be assumed that positive and negative ions are present in equal concentration, then their rate of mutual combination will be proportional to the square of the ionic concentrations. The constant of proportionality k will be somewhere between that for small ions, viz. $1{\cdot}6 \times 10^{-6}$ (Thirkhill[10]), and the approximate average value for large ions, viz. $1{\cdot}7 \times 10^{-8}$ (Hogg[7]). If some such value as

$k=2\times10^{-7}$ be assumed, and the time t taken for the ionic concentration to fall from an initial value N_0 to a value N be calculated from the usual equation $t=(1/N-1/N_0)/k$, then it is seen that with $N_0=10^5$, 99 per cent of the original particles will have combined in less than $1\frac{1}{2}$ hr.

In examining the individual observations, two points worthy of notice occur. First, the smallest values of r_0 observed on particular occasions often correspond to the existence of relatively complex aggregates, e.g. the smallest value of g observed on certain occasions may be as much as 6. Thus it would appear that, sometimes, presumably when dealing with well aged ions, the smaller nuclei have combined with each other to such an extent as to have become undetectable. Such a diminution in concentration is not unlikely from the considerations of the previous paragraph. Secondly, although it might be expected that, with a uniform source of ions, mobilities corresponding to the successive values of g would be detected, gaps in the series were often observed. This might be attributed to the variable nature of the ionization. Thus there may arrive at the place of observation air containing only the largest ions from a distant source, the smaller size having effectively disappeared by mutual combination. This air, in the course of its journey, may pass over a source of ions so near to the point of observation that these ions have not had time to combine to form many of the larger ions, and thus they would be observed as smaller ions only. Thus, the air at the point of observation may be in an apparently steady state of ionization and yet show gaps in the g series. It may be noted that inability to detect a group does not mean the complete absence of ions in that group. The minimum detectable concentration for a group of ions varies considerably with the total ion content. For the present apparatus and working conditions, the figure may be placed at from 100 to 400 per cm^3. Another factor which might conceivably result in the failure to observe successive groups is that one series of observations does not necessarily give a complete instantaneous picture of the ion population. The determination of the observational curve for a whole series of groups may occupy 2 hr. or more, but the delineation of the critical portion corresponding to a single group is usually a matter of minutes. With mixtures of ions one group, especially if present in relatively small quantity, might, over a short interval, fall in concentration to an undetectable amount, without apparently influencing the results obtained at other parts of the curve. Such changes were sometimes observed, e.g. on 13 November 1937, an observation centred at 1030 showed no detectable ions of mobility greater than 0·005, whilst about half an hour later, without any essential change of wind direction, an ion of mobility 0·0072 (not included in table 1) was suspected.

The relations between the three factors which determine the mobility of the ions in the present investigation, viz. the agglomeration factor g, the atmospheric temperature T, and the partial pressure of atmospheric water-vapour p_w, are shown in figure 7. Each of the points in this figure was obtained from the experimental results, and the curves show the manner in which the mobilities of the ions of $g=1$, 2 and 3 vary with T, when placed in an atmosphere for which p_w is constant. The observations were not sufficiently numerous to admit of many points with

identical p_w being obtained, so for constructing the curves a tolerance of ±3 per cent in p_w was given. Even with this allowance, it has not been possible to include more than a small fraction of the total observations in the diagram. From figure 7, the essential features governing the mobility of the ions may be seen. Thus, other things being equal, the mobility decreases with an increase of g, or of p_w, or with a decrease of T.

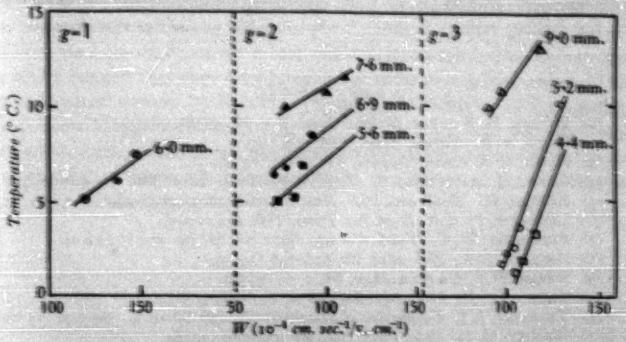

Figure 7. Variation of mobility with temperature for constant g and water-vapour pressure p_w. The figures adjoining the curves are values of p_w in millimetres of mercury.

Table 3. Average concentrations of the ions in the various size groups

g	1	2	3	4	5	6	7	8
N_g (cm³)	260	320	480	560	410	780	770	320
Obs.	7	20	14	8	11	12	4	3
g	9	10	11	12	13	14	15	>15
N_g (cm³)	920	900	520	470	600	570	640	1100
Obs.	3	2	4	3	1	3	7	12

(*b*) *Concentration of the ion groups.* If it be supposed that the $g=1$ group of ions be introduced into the atmosphere at a constant rate, and that the ions of the largest group be removed at the same constant rate, then the concentrations of the intermediate groups may be expected to assume certain equilibrium concentrations. The exact formulation of the equilibrium condition leads to cumbersome expressions. It can be seen, however, that the smaller ions, on account of their greater mobilities, would tend to have large coefficients of mutual combination. Thus, in a general fashion, it might be expected that in such a hypothetical case the concentration of ions will increase from group to group as g increases. The average figures of the concentration N_g of the ions of the successive groups $g=1, \ldots 15$, as experimentally obtained, are shown in table 3. The figures have no great theoretical significance, but there appears to be a tendency for N_g to increase with g in accordance with the above considerations.

§ 6. ACKNOWLEDGEMENTS

This work was carried out at Kew Observatory during the author's study leave, granted by the Minister for the Interior from the Commonwealth Solar Observatory, Canberra. Grateful acknowledgement is made to Sir George Simpson, F.R.S., Director of the Meteorological Office, London, for permission to work at Kew Observatory, and to Dr F. J. W. Whipple, Superintendent of Kew Observatory, for making various facilities freely available to further the progress of the work.

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DISCUSSION

Dr F. J. W. Whipple. I am glad that the Physical Society is publishing this account of one of the two investigations of fundamental importance which Mr Hogg carried out during the short period, of less than a year, which he spent at Kew Observatory. The report on the other investigation, which has resolved a baffling paradox and reconciled two methods of estimating the air-earth current, has already appeared* in Australia.

The beautiful piece of apparatus used by Mr Hogg for the work described here was constructed in Canberra and brought over to England in the expectation that intermediate ions would be numerous in the neighbourhood of London. It appears that this is the first time that the ionization 'spectrum' has been investigated in this country. That it is so definitely a sharp line spectrum is a matter of great interest.

In a recent paper† by H. L. Wright it is established that the great majority of the Aitken nuclei in urban air are so small that their presence does not affect the visibility of distant objects; the nuclei on which condensation occurs to such an extent as to produce cloud and fog are of sea-salt. These large nuclei shrink as relative

* Hogg, A. R., *Commonwealth Solar Obs. Mem.* **7**, 1939.
† Wright, H. L., *Quart. J.R. Met. Soc.* **65**, 411–439, 1939.

humidity decreases from 100 per cent to 75 per cent, but only slightly thereafter, so that variations of humidity below 70 do not affect visibility appreciably in fine weather. On the other hand, Hogg's work on mobility indicates that the intermediate ions are sensitive to humidity at any rate down to 40 per cent. This is in accordance with the idea that the hygroscopic action is due to acid. It is reasonable that nuclei formed by the evaporation of droplets of sea spray should be larger than those formed by the combination of the atmospheric gases, oxygen and nitrogen, under the influence of heat.

There has been evidence for a long while that sea-salt nuclei tend to occur in sizes which suggest that they are aggregates of units of standard weight. This has been maintained stoutly by Köhler, in spite of the difficulty of formulating any theory. Now Hogg is demonstrating that similar rules hold with the nuclei of intermediate ions. Atmospheric electricity revolutionized physics by the discovery of penetrating radiation. Is chemistry to be revolutionized by the discovery of quanta of molecular aggregation?

AN ACCURATE HARD-VALVE COUNTER CHRONOGRAPH

By F. L. UFFELMANN, M.Sc.

Received 6 *May* 1939

ABSTRACT. A 10-unit scale-of-two circuit is used to count the number of periods of a 100-kc./sec. crystal-controlled oscillator in the required time interval. The result is given instantly, correct to one period of the oscillator; the maximum error is 10^{-5} sec.

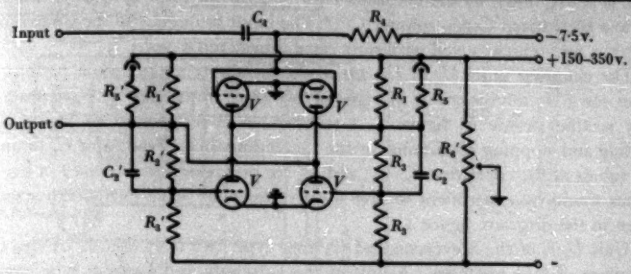
In a previous paper[1] is given an account of a chronograph, with a maximum error of 1/12,000 sec., based on a thyratron scale-of-two counting circuit. The counting speed of the thyratron circuit is restricted by the deionization time of the gas contained in the thyratron to about 20 kc./sec., and if higher counting speeds are required resort has to be had to hard-valve circuits. The present paper is a description of an improved counter chronograph, of maximum error 10^{-5} sec., based on hard-valve scale-of-two counting circuits.

Hard-valve scale-of-two circuits have been developed by Lewis[2], Stevenson and Getting[3], Lifschutz and Lawson[4, 7], Alfvén[5], and Reich[6], the circuit developed by Alfvén being identical with that of Lifschutz and Lawson. The circuits devised by these authors all involve the same principle, that of resistance cross-coupling between two valves. For such circuits there are two separate states of equilibrium in which one valve is conducting and the other non-conducting. A change from one state to the other is obtained by the application of a single impulse to the grid system.

The circuit due to Stevenson and Getting, figure 1, is sensitive to positive impulses only, but makes use of four pentode valves per unit. The circuit due to Lifschutz and Lawson, figure 2, uses two triode valves A, B per unit, but is sensitive to both positive and negative impulses, and hence needs a third triode valve D to act as rectifier between units. The circuit due to Reich, figure 3, uses two pentode valves per unit and responds to negative impulses only. An impulse of less than half a volt is required to operate it. The circuit due to Lewis (not shown) is complicated by the use of inductances, and was not considered.

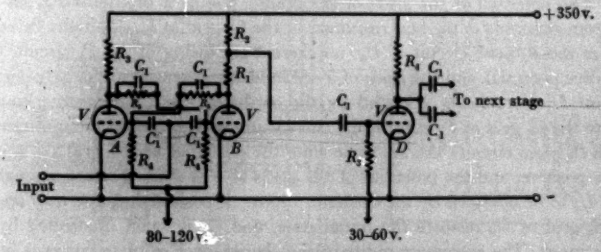
Upon test the circuits due to Stevenson and Getting, Lifschutz and Lawson, and Reich were all found to respond to a frequency of 150 kc./sec. Circuits of the first two mentioned types have been incorporated in the counter chronograph described.

The counting units. Figure 4 is a circuit diagram of the complete chronograph. The chosen input frequency of 100 kc./sec. is fed to the first counting unit U_1.



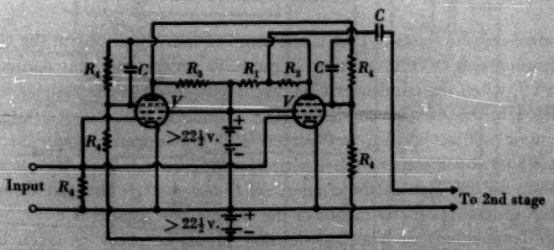
$R_1 R_1' R_3 R_3' R_4$ 100,000 Ω.; $R_2 R_6$ 300,000 Ω.; $R_5 R_5'$ 500,000 Ω.;
$C_3 C_3'$ 0·00025 μF.; C_4 0·000025 μF.; *V* Pentode type 57.
[Screen grid at 90 v.; suppressor grid at earth]
$\frac{1}{10}$ Watt neon lamps

Figure 1



R_1 20,000 Ω.; R_2 30,000 Ω.; R_3 50,000 Ω.; R_4 100,000 Ω.;
R_5 400,000 Ω.; C_1 0·000025 μF.; *V* 6C5.

Figure 2



R_1 5,000 Ω.; R_2 50,000 Ω.; R_3 55,000 Ω.;
R_4 250,000 Ω.; *C* 50 μμF.; *V* Pentode type 57

Figure 3

This is scaled down by the ten units U_1 to U_{10}, in the ratio 1024 to 1, and is then fed to a high-speed Cenco counter *CC*.* The input frequency to the Cenco counter is 97·7 c./sec. which is well within its capabilities (200 c./sec.)[8].

The counting units U_2 to U_{10} are of the Lifschutz and Lawson pattern, each using one 6N7 double triode in place of two separate triodes *A*, *B*, figure 2. The $6C_5$ rectifier triode *D*, figure 2, is retained, as it was found to insure correct starting and stopping conditions for the succeeding unit. From unit U_2 to unit U_{10} the values of the capacities C_3, C_4, and C_5 are progressively increased in size. The values found by experiment to give good rectangular wave-forms throughout are given in the diagram, figure 4.

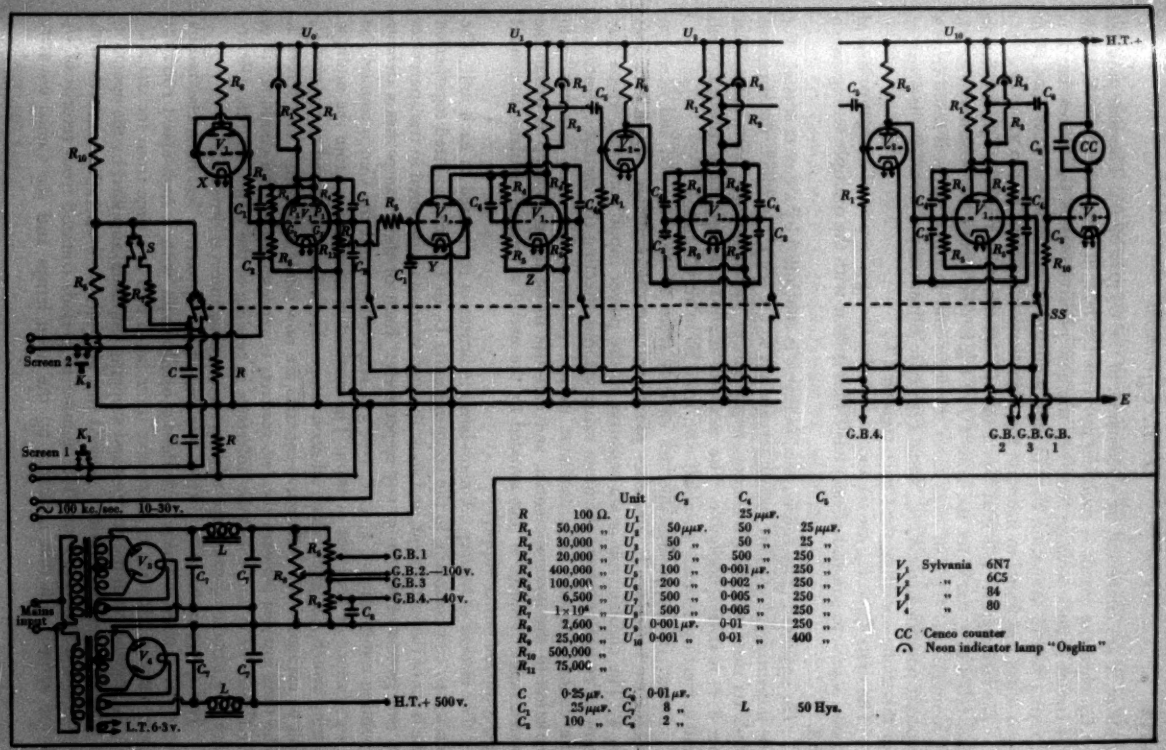
Unit U_1 is of the Stevenson and Getting type, two 6N7 double triodes taking the place of four pentodes. A unit of the Lifschutz and Lawson type would be unsatisfactory in this position, since it would be difficult to arrange for the input from the 100-kc./sec. oscillator to be admitted or blocked off by an operating unit such as U_0.

The operating unit. The operating unit U_0 is of the Lifschutz and Lawson pattern. The potential of the grids of the double triode *Y* of the unit U_1 is controlled from a tapping of the bias resistance of the G_1 grid of U_0. Initially current is flowing in the P_2 plate circuit of U_0, no current is flowing in the P_1 circuit, G_1 is at a negative potential, and the grids of *Y* are held at a potential sufficiently negative to prevent U_1 from being operated by the oscillator input. A positive impulse applied to the G_1 grid of U_0 cuts off the current in the P_2 plate circuit and transfers it to the P_1 plate circuit. At the same time the potential of the grid G_1 is made suddenly positive, and the potential of the grids of *Y* is made positive enough for the unit U_1 to be operated by the oscillator input. A positive impulse now applied to the G_2 grid of U_0 restores the initial state, and U_1 is again unaffected by the oscillator input. The extreme rapidity of the change-over of plate currents in this type of unit insures that the bias change to the grids of *Y* is of rectangular wave-form. The potential of the grids of *Y* is always sufficiently negative to prevent the flow of plate currents in *Y* when the oscillator is disconnected. If this were not so the starting and stopping impulses would be counted in addition to the oscillator impulses.

The control-impulse system. This is the same as that employed in the thyratron counter chronograph[1]. The starting impulse is generated upon contact at K_1 or "screen 1", when the condenser *C* is discharged through the resistance *R*, and is transmitted to G_1 by the condenser C_2. Similarly the stopping impulse is generated upon contact at K_2, or "screen 2". The impulse condensers *C* are charged at +83 v. by means of the multiple-setting switch *SS*. If it is required to compensate for any leakage the condensers *C* may be left charged through two one-megohm resistances R_7 by means of the double pole switch *S*.

The high-tension supply. The high-tension supply of 500 v. is taken from the valve rectifier V_4. The double triode *X* has been introduced to balance out a drop of 40 v. which would otherwise occur when the circuit commences counting. The

* Manufactured by the Central Scientific Co. of Chicago.



		Unit	C_2	C_4	C_6
R	100 Ω.	U_1		25 μμF.	
R_1	50,000 ,,	U_2	50 μμF.	50 ,,	25 μμF.
R_2	30,000 ,,	U_3	50 ,,	50 ,,	25 ,,
R_3	20,000 ,,	U_4	50 ,,	500 ,,	250 ,,
R_4	400,000 ,,	U_5	100 ,,	0·001 μF.	250 ,,
R_5	100,000 ,,	U_6	200 ,,	0·002 ,,	250 ,,
R_6	6,500 ,,	U_7	500 ,,	0·005 ,,	250 ,,
R_7	1×10^6 ,,	U_8	500 ,,	0·005 ,,	250 ,,
R_8	2,600 ,,	U_9	0·001 μF.	0·01 ,,	250 ,,
R_9	25,000 ,,	U_{10}	0·001 ,,	0·01 ,,	400 ,,
R_{10}	500,000 ,,				
R_{11}	75,00C ,,				

C	0·25 μF.	C_5	0·01 μF.		
C_1	25 μμF.	C_7	8 ,,	L	50 Hys.
C_3	100 ,,	C_8	2 ,,		

V_1	Sylvania	6N7
V_2	,,	6C5
V_3	,,	84
V_4	,,	80

CC Cenco counter
Neon indicator lamp "Osglim"

Figure 4. 10-stage counter chronograph.

grids of X are connected by a resistance to the G_2 grid of U_0. The bias provided by G_2 when the circuit is counting prevents the flow of plate current in X. When the circuit is not counting, however, the now positive bias of G_2 insures that X is carrying its full saturation current, and adjustment of the anode resistance R_6 to 6500 ohms was found to maintain the high-tension constant at 500 v.

The bias supply. The grid bias is taken from the valve rectifier V_3. A common bias GB 2 of -100 v. for the units U_0 to U_{10} is taken from the potentiometer R_8. A common bias GB 4 of -40 v. for the inter-unit rectifier valves is taken from a separate potentiometer $R_5 R_9$, while a separate adjustment of bias GB 1 is provided for the triode operating the Cenco counter *CC*.

The units U_0 to U_{10} are set successively by the application of a negative bias of about -40 v. to one grid of each unit. This is done with the multiple setting switch *SS*.

The oscillator. The 100 kc./sec. oscillator is controlled by a quartz crystal. For the measurement of short time-intervals (up to 0·1 sec.), for which the apparatus finds its chief use, the {frequency, temperature} correction of 5 parts per million per 8° F. is always negligible. The oscillator input to the apparatus is not critical, and correct operation is obtained with from 10 to 30 v. r.m.s.

Power. The apparatus operates entirely from an a.-c. supply, and needs no subsidiary batteries. It consumes 300 w. without the oscillator, which consumes 25 w. The chronograph is not sensitive to the power input potential, which may vary by 10 per cent from the mean value without affecting the correct operation of the counter circuits. The oscillator output potential, however, is sensitive to variation in the power potential, and care has to be taken to keep it within the stated range. Trouble is thus avoided by the use of a voltage regulator to control the complete power supply.

The method of reading. A neon-lamp indicator is connected to each unit. The ten lamps corresponding to the counting units U_1 to U_{10} are mounted together on the front of the instrument, and show through windows numbered in transparencies in two groups of five, 1–5, 1–5 respectively. Each of these lamps has a definite counting significance 1, 2, 4, ... 256, 512 respectively, and the Cenco counter has a significance of 1024 per division. The method of use is to operate the multiple setting switch *SS*, after which all neon lamps except that connected to the unit U_0 will be extinguished. Successive contacts are now made at screen 1 and screen 2. The time interval between the two contacts is represented by the combination of the lighted neon lamps and the difference in reading of the Cenco counter. Reference to a suitably constructed table gives the time interval directly.

Test of apparatus. The correct working of the apparatus has been checked by means of a triple-element cathode-ray oscillograph with a high-speed rotating-drum camera. Records of successive units taken two at a time from the units U_1 to U_{10}, together with the starting and stopping impulses, supplied by the operation of a telephone selector switch, showed that all units were operating correctly with good rectangular wave form. The number of periods counted from records of U_1 always agreed with that indicated by the neon lamps and the Cenco counter.

Records of U_1 and the control impulses also showed that the maximum error of the apparatus is one period of the oscillator, i.e. 10^{-5} sec.

Higher input frequencies may be used if greater accuracy is required. Tests have been made at 150 kc./sec., when the Cenco counter responds correctly at 146·5 c./sec.

Use of the apparatus. The apparatus has been designed primarily for the rapid and accurate measurement of time intervals up to 0·1 sec., but is equally suitable for longer time intervals. If preferred, photoelectric operation can be used instead of operation by contact. The impulse condensers C are then discharged by thyratrons of the 885 type.

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AN ACHROMATIC OBJECTIVE FOR USE IN ULTRAVIOLET MICROSCOPY

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Received 19 *June* 1939

ABSTRACT. The design of a low-power achromatic microscope objective for use in the ultraviolet region is described. By employing lithium fluoride combined with fused quartz, achromatism of the lens system is obtained for a considerable range of the visible and ultraviolet spectrum. This greatly facilitates the technique of ultraviolet microscopy, for it is only necessary to focus the object in visible light and the ultraviolet will be simultaneously in focus. The use of certain ultraviolet filters with a modern high-pressure mercury-vapour lamp is also discussed.

§ 1. INTRODUCTION

In a letter to *Nature*[5] the writer has mentioned the design of a microscope objective composed of lithium fluoride[1] and fused quartz, which was to be achromatic over a considerable range of the visible and ultra-violet spectrum. This objective has now been made by Messrs W. Watson and Sons, Ltd. in accordance with the design and has been tested. The results form the subject of this paper.

§ 2. THE DESIGN

The primary consideration in the design was an attempt to produce a lens system which could be used for focusing the object in visible light, such as that corresponding to the mercury line 5461 A., and which would have precisely the same focus for one or more ultraviolet wave-lengths. Thus the necessity of finding the ultraviolet photographic focus (hitherto an inherent difficulty with monochromats) would be entirely eliminated and the technique of ultraviolet microscopy greatly simplified. In such an attempt it was considered advisable to confine the design to a low-power objective, and therefore nothing more ambitious than a nominal 25-mm. focal length with numerical aperture 0·20 was decided on. The optical design was carried out by the usual Conrady methods, namely a solution by algebraic approximations for the chromatic aberration, spherical aberration, and coma as given by various bendings of the lens system, followed by strict trigonometrical ray-tracing. As the optical constants of lithium fluoride may not be

familiar to some readers, they are set out, together with those of fused quartz, in the following table:

	Wave-length (A.)				$\frac{N_{4048}-1}{N_{2749}-N_{5461}}$
	5461	4048	2749	2313	
Lithium fluoride	1·3929	1·3983	1·4128	1·4244	20·0
Fused quartz	1·4602	1·4697	1·4961	1·5194	13·1

It was decided to achromatize the lens system for wave-lengths 5461 and 2749 A. and to correct the spherical aberration for the latter wave-length. The calculations by the analytical method give the following values for spherical aberration and coma for various shapes of the lens system:

Curvature of contact surface. $1/r_2$	−0·10	−0·15	−0·20	−0·25	−0·30
Spherical aberration	+0·330	+0·015	−0·155	−0·181	−0·062
Coma	+0·0048	+0·0031	+0·0014	−0·0003	−0·0021

If the parabola for spherical aberration is plotted simultaneously with the coma curve, two solutions will be found for zero spherical aberration, but they have a coma value outside the permissible tolerance. By allowing full tolerance to the spherical aberration, however, the coma can be reduced to a value nearly within the tolerance (namely ±0·0025). The most suitable solution chosen from these graphs is used as a basis for the trigonometrical tests. The ray-tracing procedure was that usually adopted for microscope objectives, namely that of tracing from the long conjugate distance to the short conjugate distance. The optical tube-length is 160 mm. and the primary magnification 6·4.

The final solution gives a lens system of the following specification:

$$r_1 = 7{\cdot}937 \text{ mm.}, \qquad r_2 \equiv r_3 = 6{\cdot}372 \text{ mm.}, \qquad r_4 = 1887 \text{ mm.},$$

where r_1 is the radius nearest the image, r_2 and r_3 are the contact radius, and r_4 is the radius nearest the object. Radii r_1 and r_2 refer to the lithium fluoride lens and r_3 and r_4 to the lens of fused quartz. Their respective axial thicknesses are 4·0 and 2·0 mm. A thin film of glycerine may be used between the contact surfaces. As r_4 is such a long radius, a plane surface may be substituted for the sphere without appreciable effect on the aberrations; this also simplifies the optical manufacture. There is no particular difficulty in producing optically worked surfaces in lithium fluoride, but it may be of interest to record that magnesium oxide acts rather better than rouge as a polishing medium.

Ray-tracing tests of the chromatic aberration gave a residual value of −0·008 mm. for 5461 and 2749 A. The tolerance $\pm\lambda/N^1 \sin^2 U^1_m$ is ±0·0194 mm. Other wave-lengths were tested trigonometrically, namely 4861, 4048 and 3651 A., and it was found that the residuals were all within their respective tolerances. Figure 1 shows these values plotted; it illustrates the exceptionally small secondary spectrum obtainable when lithium fluoride is used together with fused quartz, and the consequent suitability of a combination of these two materials in an achromatic lens.

It will be seen from the foregoing that as regards colour-correction, this lens is in effect apochromatic, although according to Abbe's original definition of the term[6] such a lens must be spherically corrected for two wave-lengths in addition to being chromatically corrected for three wave-lengths. Nevertheless, whilst this lens is spherically corrected for one wave-length only, it is chromatically corrected over a greater range of wave-lengths than previously known apochromatic lenses. This is due primarily to the small dispersion values of the newly introduced lithium fluoride and of fused quartz.

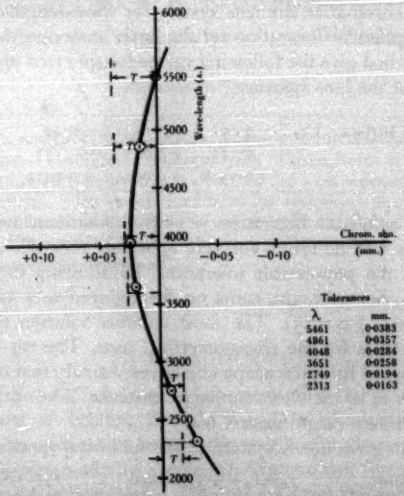
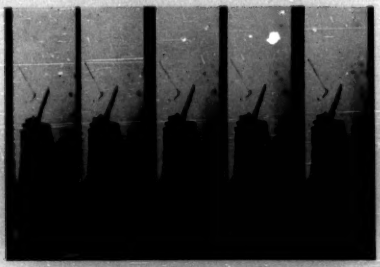


Figure 1. Chromatic aberration curve for lithium fluoride, fused quartz, objective

The practical importance of the results shown in figure 1 is considerable, for it means firstly that no focusing adjustment of the microscope is necessary when the change from visual illumination to ultraviolet illumination is made, and secondly that a substantial range of ultraviolet wave-lengths may be employed without seriously affecting the definition of the image. The latter fact enables a much greater intensity of illumination than was hitherto practicable to be used with the ultraviolet microscope, with a consequent reduction in exposure of the photographic plate. In fact it has now been found possible to see the ultraviolet image on a fluorescent screen placed in the plane of the plate, whereas previously the low luminosities entirely precluded this.

§ 3. TESTS

The preliminary tests consisted in (1) the determination of the correct tube-length, an artificial star in the form of a silver-film slide being used as object, and (2) a numerical test of the resolving power. The latter, as measured by a method



λ 5461 A. λ 4358 A. λ 3650 A. λ 2749 A. λ 2313 A.

Figure 2. Magnification, 62 diameters.

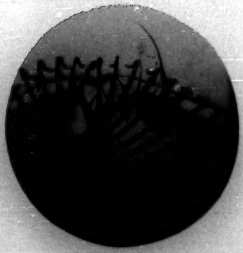


Figure 3. Magnification, 62 diameters; $\lambda = 2749$ A. only

Figure 6. Magnification, 62 diameters; spectral range, 4000 to 2550 A.

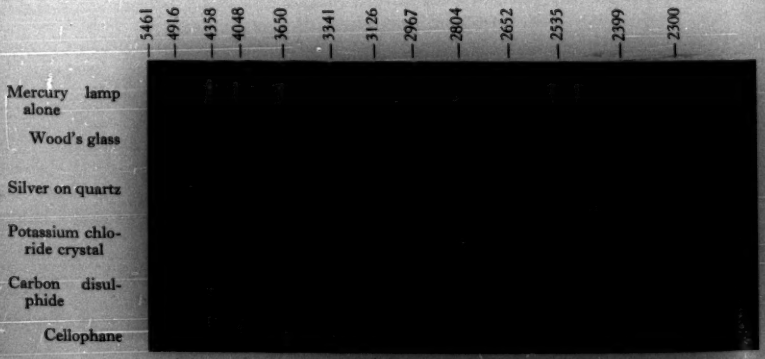


Figure 5

To face page 1037

already described[a], had a value of 1·12 μ. The theoretical resolving-power of this lens system for 2749 A. is 0·82 μ., and the necessary magnification for photomicrographic work is $\times 61$.

The test for chromatic aberration was then made. For this purpose, the objective was focused on a thin* flat object, actually a silver film on a quartz slide, and monochromatic green light of wave-length 5461 A. was used. By means of a quartz monochromator placed in front of the microscope, light of wave-lengths 4358, 3650 and 2749 A. was brought in turn on to the substage condenser; and without the fine adjustment being touched in any way, photographs were taken. The first four pictures (from left to right) in figure 2 show the result, and the sharpness of the image in each case indicates that the focus has not changed appreciably. As a matter of interest, wave-length 2313 A. was also used, still without alteration of the fine adjustment, and a photograph (the last picture on right of figure 2) was taken; this is definitely out of focus, as is to be expected from the curve in figure 1. If, however, a slight focusing adjustment was made, the definition given by the lens at 2313 A. was still quite satisfactory. For those who are more familiar with definition tests in which natural objects are used, a frequently used test object for low-power objectives (namely the proboscis of the blow fly mounted on a quartz slide) has been photographed, figure 3, with the lens at 2749 A. Some idea of the definition achieved can be obtained from this, although the less interesting silver-film object is really a better test object.

As has already been mentioned, the almost apochromatic nature of this lens system lends itself to the use of a considerable spectral range for illumination purposes, and consequently we may combine the ultraviolet spectrum between 4000 and 2500 A., by methods which will be indicated later, and we thus obtain a resultant increase in luminosity of the image. This fact is of particular interest at the lower end of the ultraviolet spectrum (i.e. the region 2000 A.) where intensities are generally much lower; for one can combine a group of lines (e.g. the cadmium group 2313, 2265, 2195, 2144; the zinc group 2138, 2100, 2062, 2026; or the aluminium group 2094, 1990, 1863, 1855) and still maintain good definition whilst having available a much greater intensity of illumination than has hitherto been attainable with other lens systems in the ultraviolet region.

§ 4. ILLUMINATION SYSTEMS

Two methods may be employed for combining the whole or individual parts of the ultraviolet spectrum for use in the microscope. The first is the use of a monochromator arranged as in figure 4, in which a slightly dispersed ultraviolet spectrum is formed at S; a quartz lens L_2 then projects a reduced image of L_1 on to the substage condenser. By the placing of narrow opaque strips in the spectrum at S, or by movement of the monochromator, any desired part of the spectrum may be removed, while the remaining part will be transmitted and be finally combined by the lens L_2 in the microscope.

* That is, thin compared with the depth of focus of the objective.

Alternatively, an ultraviolet source of light of high intrinsic brightness may be used together with suitable ultraviolet filters, which are now available. A convenient source is one of the modern high-pressure mercury-vapour lamps.* The use of filters with an ultraviolet lamp is only possible because the lens is chromatically corrected for a considerable range of wave-lengths, and because of a recent process devised by R. W. Pohl[3, 4] which enables such ultraviolet filters to be made. By raising certain crystals to a suitable temperature and then subjecting them to electronic bombardment, it is possible to produce selective absorption in certain parts of the spectrum. R. W. S. Garton, of the Imperial College of Science and Technology, has reproduced some of Pohl's methods, and I am indebted to him for providing me with a potassium-chloride crystal which has been treated accordingly.

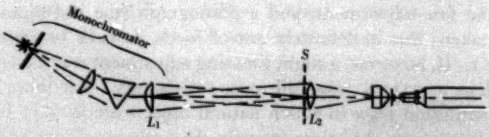


Figure 4

In this connexion the transmission characteristics of certain filters available for ultraviolet work may be of interest. Figure 5 shows a number of these.

	Thickness (mm.)	Transmission (A.)
(*a*) Wood's glass	11	4000 to 3100
(*b*) Silver on quartz	Densely silvered	4400 to 2950
(*c*) Treated KCl crystal	3	4000 to 2300
(*d*) Carbon disulphide in quartz cell	0·5	7000 to 3600 and 2900 to 2650
(*e*) White cellophane	0·32	7000 to 2550

Photomicrographs have been taken by using the high-pressure mercury lamp directly, together with a combination of the various filters set out above and with the lens described above, but space does not allow of the reproduction of all of these. It was felt, however, that the photograph taken when filters (*c*) and (*e*) were used together would be of interest, for it represents the normal procedure with this lens, namely that of focusing with a glass filter which transmits visual green light and then replacing this by a combination of ultraviolet filters (*c*) and (*e*). Figure 6 illustrates the definition obtained when a range of the ultraviolet spectrum, in this case from 4000 to 2550 A., is used.

Obviously by employing filters (*c*) and (*d*) a more limited transmission, between 2900 and 2650, may be obtained if desired. It is hoped that other treated crystals may shortly lead to the production of further limited transmissions in the region from 2200 to 1850 A.

* The one employed by the writer was supplied by the General Electric Co. Ltd. and operates at 230 volts a.c. with a suitable choke. The external glass envelope was specially removed for the purpose in view.

§ 5. CONCLUSIONS

The claim that the lens system fulfils the purpose for which it was designed has been fully justified in practice, and it would appear that the technique of ultraviolet microscopy must be simplified by means of the system. It is, of course, desirable at this stage that high-power lenses of this type should be designed and made. It might be advisable, in future, to achromatize the lens system for the blue end of the visible spectrum, instead of for green light, and for some ultraviolet wave-length shorter than 2749 A., such as 2400 A. for instance, with the result that the lens might be achromatic throughout the entire range from 4000 to 2000 A.

It will be realized, of course, that both the photomicrographs and the spectrograms lose definition and contrast during the course of reproduction, but the negatives are always available for those who wish to see them.

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OBITUARY NOTICES

PROFESSOR FREDERICK JOHN CHESHIRE, C.B.E., A.R.C.S.

F. J. CHESHIRE was born at Leeds on 8 June 1860. He came to London after leaving school with a National Scholarship at the Royal College of Science, and obtained the diploma. He also attended classes at the Birkbeck Institute (now Birkbeck College). He was for a short time in the works of Greenwood and Batley of Leeds. In 1880 he entered the Customs and Excise, but in 1885 he joined the Staff of the Patent Office, which two years earlier had undertaken the skilled examination of patents instead of their mere registration. As an Assistant Examiner he was concerned with scientific instruments. In 1915 he realized the importance of the supply of scientific instruments in the conduct of the war, and he called on Sir Eric Geddes to impress this importance upon him, with the result that Sir Eric obtained his transfer to the Ministry of Munitions. There he was made Deputy-Director General of the Optical Department. In 1917 he was appointed Director of the Optical Engineering Department of the Imperial College, South Kensington, where he remained till his retirement in 1925. He was awarded the C.B.E. in 1918. He was a great admirer of Abbe and of the Carl Zeiss organization which he visited more than once; he contributed three notes on Abbe letters to the Royal Microscopical Society. His contact with Zeiss and especially with Prof. von Rohr had convinced him of the importance of optical computation, and was no doubt the cause of the appointment of Prof. Conradi to the staff of the college to develop that side of the teaching—an appointment of great value to the development of the department. He was President of the Optical Society from 1916 to 1920 and of the Royal Microscopical Society in 1922 and 1923. His presidential addresses to the latter were upon "The early history of the polariscope and of the polarizing microscope" and upon "The design of the petrological microscope".

I first met Cheshire when he was an Examiner in the Patent Office and was sharing with F. Boughton the teaching of physics at the Birkbeck Institute, Cheshire lecturing on Friday evenings for 2 hours to the elementary students while Boughton took the advanced classes on Monday and Wednesday evenings. In those days the old Science and Art Department paid on the results of the examinations; £1 for an elementary pass, £1. 10s. for an advanced pass and £2 for an honours pass. The classes were farmed out to the teachers who took the money gained by the passes of their students. After about 1897 the Birkbeck Institute paid a fixed salary to the teachers. These lectures were attended by students, both men and women, working for degrees, and for other examinations such as those of the Patent Office. Cheshire began helping Boughton in 1893 and continued to take the elementary physics up to 1911. The lectures were very fully illustrated by experiments, and Cheshire was fertile in ideas for new experiments, especially in sound and light, in which he was

always most keenly interested. About 1898 or 1899 he was asked by the London County Council to give a short course of about six lectures to men in the Piano industry. These were given in the Camden School of Art. Some 80 men attended, and after he had made it clear that he did not pretend to know anything of the making of a piano, but only of the principles involved, they became very keen. The lectures were repeated a second year. The lectures were largely experimental, and illustrated by many original experiments carried out with simple apparatus which Cheshire devised and made himself. When he took up any subject such as this, Cheshire became full of enthusiasm; he thought and talked of it at every opportunity, and spared no trouble to make it a success. I remember his giving a lecture on "The spheroidal state" at a Conversazione at the Birkbeck Institute, and among the experiments both he and his son, then a small boy, ladled molten lead out of a plumber's pot with their naked hands on to the floor; incidentally, he spoilt a new pair of trousers by the splashing of the lead on to its legs. Another subject which he took up in the same whole-hearted way was that of polarized light. He made and bought a lot of special apparatus for it and did many beautiful experiments in his lectures. At a later date at the Imperial College he revived this and lectured on it to the minerological students. In his later years he turned his enthusiasm to golf, and designed a well-known club that could be specified according to the angle of its face.

In 1886 he married Mary, daughter of Mr George Richardson of Huddersfield. They had the son Ralph above referred to, and one daughter. They lived at Carson Road, Dulwich, for most of their married life, but about a year ago moved to be near their daughter, now married and living at Reading.

Prof. Cheshire made a number of contributions to the Optical Society and also to the *Optician*, and he wrote a pamphlet upon the optical identification of crystals for his students at the Imperial College, but his chief contribution was rather as a leader and organizer than as a scientist. His wonderful energy and enthusiasm for everything he undertook infected his colleagues and helped in the progress that resulted, and it laid the sound foundation of the necessarily small but none the less important department which is now in charge of Dr L. C. Martin.

R. S. C.

MR J. EDGAR CRACKSTON, M.SC.

His many friends will have heard with deep regret of the death of Mr J. Edgar Crackston, lecturer in physics at Cardiff University College since 1923. He had nearly completed a year's residence in America as an exchange lecturer when he was overtaken by serious illness and returned home. He died at Penarth on August 17, 1939, at the age of 45.

Mr Crackston entered Liverpool University in October 1914. Eighteen months later he left to join the army, and saw active service in France. Returning to Liverpool he completed his degree course with first class honours in physics, and was awarded an Oliver Lodge Fellowship. For his researches at Liverpool under

Prof. Wilberforce (1920–21) and at Cambridge under Prof. Sir J. J. Thomson (1921–3) he obtained the M.Sc. degree of both these universities.

Those who knew him intimately will long remember him for the nobility of his character. He set himself a high standard, both in his private life and in his profession, and never failed to live up to it. He is survived by a widow.

R. T. D.

SIR FRANK DYSON, K.B.E., F.R.S.

ASTRONOMY has suffered a great loss by the death of Sir Frank Dyson, ninth Astronomer Royal, which occurred on 25 May 1939 whilst he was on a voyage from Australia to South Africa. Although primarily devoted to astronomy, he was necessarily interested in the numerous applications of physics in modern astronomy, and was not only a Fellow of the Physical Society and of the Institute of Physics, but served as President of the Institute during the period 1927–9.

Frank Watson Dyson was the son of the Rev. Watson Dyson, a Baptist minister, and was born at Ashby on 8 January 1868. He was educated at Bradford Grammar School and Trinity College, Cambridge, and was second wrangler in the Mathematical tripos in 1889. His subsequent election to one of the Isaac Newton studentships founded by Dr Frank McClean gave him every facility for entering into astronomy as a career.

In 1894 Dyson was appointed Chief Assistant at the Royal Observatory, Greenwich, when Turner had left to become Savilian Professor at Oxford. In 1905 he succeeded Copeland as Astronomer Royal for Scotland, and five years later returned to Greenwich as Astronomer Royal on the retirement of Sir William Christie. He retained this high office until his retirement under the age limit in 1933.

During his first period at the Royal Observatory, Dyson was closely associated with the work on the Greenwich zone of the International Astrographic Chart, and with the successful determination of the solar parallax from photographic observations of the planet Eros at the opposition of 1900–1. What is generally regarded as his most important contribution during this period, however, was a new reduction, made in collaboration with Mr Thackeray, of the Groombridge Catalogue of Circumpolar Stars, and the consequent determination of the proper motions of 4250 stars; this provided most valuable data for the discussion of stellar movements.

Whilst at Edinburgh he took up, among other investigations, the question of star streaming, and showed that Kapteyn's results were confirmed, especially by stars having large proper motions.

On Dyson's return to Greenwich in 1910, remarkable progress was made with the astrographic catalogue, and a programme of determinations of the parallaxes of stars situated in the Greenwich zone was initiated. It has been said that there is probably now no part of the heavens for which our knowledge of the positions, proper motions and distances of the stars is so complete as that of this zone, extending from $+64°$ to the Pole. Under Dyson's direction, advantage was taken of improvements in instruments and methods of observation, and the work on

meridian observations, variation of latitude, double stars, sun spots, terrestrial magnetism, stellar photometry and the time service fully upheld the high reputation of Greenwich among the observatories of the world. The familiar device of six pips adopted for the broadcast time signals was originated by Dyson.

Whilst Dyson's energies were mainly devoted to positional astronomy, he made substantial contributions to astrophysics by his spectrographic observations during total eclipses of the sun, beginning with that in Portugal in 1900. He earned the reputation of being capable of controlling the weather on such occasions, having made successful observations on each of his five expeditions. His early discussions of the flash spectra which he photographed were notable for his confirmation of Lockyer and Fowler's conclusions as to the importance in this connexion of what are now recognized as lines of ionized atoms, which had met with considerable opposition from several eminent astronomers. A contribution of permanent value to the practical and theoretical aspects of eclipse observations is afforded by Dyson and Woolley's book on *Eclipses of the Sun and Moon*, published in 1937.

It was Dyson who first pointed out that the total eclipse of May 1919 would afford a particularly favourable distribution of stars in the neighbourhood of the sun for putting to the test Einstein's prediction of the deflection of light by the sun's gravitational field. In spite of many difficulties due to the war, he persisted in the organization of expeditions to Brazil and Africa, on the chance that it might become possible for them to set out when the time arrived. The expeditions were fortunately able to proceed and although Dyson himself was unable to accompany either of them, his foresight was amply rewarded by the historic verification of Einstein's prediction.

Dyson took a full share in the general promotion of astronomy at home and abroad. He was deeply interested in the organization and welfare of the International Astronomical Union, of which he was President in the years 1928 to 1932; and in the affairs of the Royal Astronomical Society, in which he was successively Secretary, President and Treasurer. Among other outside activities, he took an important part in the establishment of the University of London Observatory at Mill Hill; he was chairman of the Board of Management, and the acceptance of the fine Oxford 24-in. refractor as a gift from the Radcliffe Trustees was largely due to his influential support.

Dyson was knighted in 1915 and created K.B.E. in 1926. Honorary degrees were conferred upon him by the Universities of Cambridge, Oxford, Edinburgh, Leeds, Perth, Melbourne, Durham and Toronto. He was elected a Fellow of the Royal Society in 1901, and was awarded a Royal Medal by that body in 1921. He received the Bruce Gold Medal of the Astronomical Society of the Pacific in 1922 and the Gold Medal of the Royal Astronomical Society in 1925. In 1928 he was the first recipient of the Gold Medal of the British Horological Institute. He was a Foreign Member of the Reale Accademia Lincei and of the American Academy of Arts and Science; and a Corresponding Member of the Academy of Sciences, Paris, the Bureau des Longitudes, Academy of Sciences, Russia, Royal Academy of Sciences, Belgium, and the Spectroscopic Society of Italy.

Dyson married Caroline, daughter of Palemon Best, M.D.; she died in 1937, leaving two sons and six daughters. He was one of the most amiable and unselfish of men, and will long be held in affectionate remembrance by all who served under him or had the privilege of his friendship.

A. FOWLER

MR R. KANTHACK

RALPH KANTHACK, who died on 12 November 1938, was elected a Fellow of the Optical Society in 1924. Born in 1861, the son of the British Consul of Para, Brazil, he was educated at Hamburg and Lüneberg, and thus acquired that deep and colloquial knowledge of German which he later used to such advantage in his translation of technical works on his chosen study, optics, both from and into German. He had been associated with Zeiss and with Adam Hilger, for the latter of whom he prepared the valuable table of refractive indices. His other publications included a translation of Heath's *Geometrical Optics*, and a Government publication on *The Theory of Optical Instruments*. He leaves a widow.

LOUIS LOWNDS

LOUIS LOWNDS was born 24 August 1876 at Repton, Derbyshire. At the exceptionally early age of 17 he began work as a science teacher in St Andrew's School, Derby, and two years later was appointed an evening lecturer in the Municipal Technical College of that city. In June 1897 he gained the Heymann Scholarship at University College, Nottingham, where he assisted Dr E. H. Barton in his researches on the transmission of electric waves along wires. Meanwhile he worked for the B.Sc. degree of the University of London, which he took with honours in experimental physics in 1898. In the following year he continued his work with Dr Barton, and also carried on research with Dr S. W. Richardson, on the magnetic properties of alloys of cast iron and aluminium. These years of research impressed the authorities at University College, and in 1899 when it became their privilege to recommend an 1851 Exhibition Scholar, Lownds was selected for that honour. He elected to hold the scholarship at the Royal University of Berlin, where he worked for three years under Prof. E. Warburg. The subject of study assigned to him was the experimental examination of the thermomagnetic and thermoelectric properties of a single bismuth crystal, which was supplied most kindly by Prof. Groth, the distinguished mineralogist of Munich. It is interesting to reflect that this study of the properties of a single crystal in contradistinction to those of an aggregate of crystals has become in recent years a matter of fresh importance in connexion with strength and structure of materials.

On returning to England in 1902 he was appointed demonstrator of physics at the Chelsea Polytechnic, in the department of mathematics and physics under Dr W. H. Eccles. Six years later the department was separated, and Dr Eccles took

charge of mathematics, leaving physics to Dr Lownds. At the same time the laboratory was moved to the new north wing, where an instrument shop with a mechanic were added. Dr Lownds now had full charge with his assistants of all the classes in experimental work, and he devoted all his energy to teaching students, from beginners up to candidates for B.Sc. honours. In 1914, when the world war broke out, Dr Lownds gave much time to work for the War Office, and after the peace of 1918 he welcomed the large increase in students which arose through the demand for training.

Dr Lownds took a prominent part in the Physics Board of the University of London and he acted as examiner in all branches of his subject for many years. He was the author of *A First Book of Physics*, published by Macmillan and Co. He joined the Physical Society in 1903, and was a Founder Fellow of the Institute of Physics formed in 1920.

Late in 1937 his health broke down, and in January 1938 he was given sick leave. Unfortunately there was no improvement, and he retired at the end of the session. He did not live long, and in September 1938 he died at the age of 62.

Dr Lownds will be remembered by a host of students, both day and evening, for his extremely clear teaching. He took pains to illustrate it with lecture demonstrations wherever possible, and for this purpose he designed and constructed a very full set of apparatus. He helped many advanced students to commence research, for which his laboratory became well known, and many such students owe a deep debt of gratitude to him.

S. S.

ALFRED WILLIAM PORTER, D.Sc., F.R.S.
Fellow of University College, London

Alfred William Porter was born on 12 November 1863. After attending school he went into an architect's office; this accounts for the neatness and beauty of his diagrams, in which he took justifiable pride. It was thus somewhat later than is usual that he took up a scientific career, first studying at Liverpool under Oliver Lodge. In 1889 he came as a student to University College, London, and took his degree of B.Sc. in the University of London. In 1891 he received an appointment on the Physics Staff at University College, and he became Assistant Professor in 1896. In the latter year he was elected a Fellow of University College and in 1911 a Fellow of the Royal Society. In 1912 the University made him a Reader in Thermodynamics, and in 1924 he became Professor in the Department. In 1928, having reached the age of 65, he retired with the title of Professor Emeritus. He now gave a considerable time to the editorship of the *Philosophical Magazine*. Latterly the decline in health of both mind and body was painfully apparent to those who met him often. A fall he had a couple of years ago accelerated, but did not start, this failure of health. He had no relations in London, and lately moved to West Kirby to be near his own people. He died there shortly after his move on 11 January 1939.

The above bare record only touches the fringe of Porter's activities. In the

College itself he took an active part in the foundation of the Union and Critical Societies. In the University he was for many years Hon. Secretary of the Board of Studies in Physics. He was a Founder Fellow of the Institute of Physics and its first Hon. Secretary from 1919 to 1926, President of the Röntgen Society in 1913–14, and President of the Faraday Society from 1920 to 1922, of which he was a Founder Member. He was Recorder of Section A of the British Association in 1906–12, and President of that Section at the Glasgow Meeting of 1928. He accompanied the British Association to Australia, and on that occasion he was made a D.Sc. of Melbourne University, *honoris causa*.

Porter was too much occupied in his academic and other duties to undertake much experimental work himself, but he would on occasion indulge in it. Thus, when Röntgen first published his discovery of X-rays, he tried all the vacuum bulbs in the Department. One of these, with an anticathode, gave sharp images, much in advance of anything shown till that time. This was exhibited at a public lecture and thus became widely known. Again, his theoretical deductions pointed to the fact that a thin layer of a moderate non-conductor of heat might actually lead to the cooling of the heated body. This was strikingly illustrated in a uniform platinum wire heated to some 800° C. and covered along its length with different thicknesses of glass. But his chief work was undoubtedly on the theoretical side. This is not the place to enumerate his many papers in dictionaries and periodicals; reference will be made only to the books with which his name is connected. In 1903 was published, under the names of Foster and Porter, the second edition of a treatise on electricity founded on Joubert's work. This contains much original matter from Porter. In 1905 he published a book on intermediate mechanics, in which the idea of inertia is stressed, rather than that of force. The fifth edition of Preston's *Light* (1928) was edited by him, while in 1931 Messrs Methuen issued his *Monograph on Thermodynamics*, and in 1933 in the same series *The Method of Dimensions*.

It is difficult to say in which branch of physics Porter took most interest. As some obscurity occurred to him, he would work at it assiduously until satisfied. Thus, some thirty years ago, he took up the question of the seat of the electromotive force in the Volta effect. This investigation was made use of in his address at Glasgow in 1928. In later years he had become much interested in capillarity, the more so because he was convinced that the anomalous results observed in capillary tubes in the neighbourhood of the critical point were due to inadequate corrections for pressure and volume, to say nothing of ignorance of the magnitude of these corrections. Founding his formulae on Bashforth and Adams's results, however, Porter published tables in convenient form enabling all necessary calculations to be made for tubes and sessile drops of any diameter. Needless to say, he found that papers already published did not give all the data required, nor, in one particular case which he was anxious to investigate, was the original apparatus recoverable. Thoroughness was perhaps his greatest characteristic. He disliked expressing an opinion until he had thought it over at his leisure. Hence the occasional obscurity of his answers to students after a lecture; but they were amply rewarded in a day or two by having the problem completely worked out. Now and again he would be

goaded into an indefensible statement, which he would proceed to defend with all the dialectical skill at his command. Then a deprecatory wave of the hand or a merry twinkle in his eye would terminate the discussion. He was indefatigable in his desire to help researchers in his own or in other departments, and his correspondence testified to the help he was able to give.

Those of us who knew him well will long regret the passing away of one on whose advice they could rely.

N. E.

FRANK CHARLES WATTS

MR FRANK CHARLES WATTS, Managing Director of E. R. Watts and Son, died on 24 March 1939 at the age of 68. He was born in London on 30 October 1870, the second son of E. R. Watts, the founder of the well-known firm of surveying-instrument makers.

Frank Watts and his brother George—who later developed a very accurate dividing engine—served their time as apprentices in their father's workshop. In those days the hours spent in the factory were long, both boys working from 6 a.m. to 6 p.m., with a considerable amount of overtime added to the normal workshop hours. On the completion of his apprenticeship Frank began to relieve his father of the managerial side of the business, eventually taking charge after his father's death in 1901. In 1919 the firm was converted into a limited liability company with Frank Watts as Chairman. Under his direction the firm grew rapidly, becoming known for the manufacture of theodolites and levels of efficient design and sound workmanship. It was however in his outlook on export trade and its possibilities that Frank Watts showed his special characteristics. He was a great believer in taking his wares to the customer, and under his direction a member of his staff exhibited and demonstrated Watts's instruments to the surveying departments of the majority of foreign and colonial governments.

Although the name of Frank Watts is not associated with any particular development in surveying instruments, he frequently made pertinent suggestions concerning their design. His workshop training enabled him to suggest methods for the production of instruments in large quantities, a most useful gift when the question of making large numbers of instruments for government departments arose. He had served almost continuously, since its formation in 1916, on the Council of the Scientific Instrument Manufacturers Association (originally the British Optical Instrument Manufacturers Association), where his keen business judgment was much valued. He was elected a Fellow of the Optical Society in 1914 and served on its Council from 1918 to 1921, and 1924 to 1926; he acted as Vice-President from 1927 to 1930.

He was most modest as to his own capabilities but was greatly trusted and liked by his colleagues and friends, and by the men working under him. He leaves a widow and daughter.

R. S. W.

DR A. E. H. TUTTON, F.R.S.

ALFRED EDWIN HOWARD TUTTON, who died on 14 July 1938, was born on 22 August 1864 at Stockport, Lancashire. He entered the profession of science through evening classes at the Stockport Mechanics' Institute, which he attended whilst employed in a solicitor's office. Next he obtained a scholarship which enabled him to study at South Kensington, and in 1887 he became assistant demonstrator in chemistry. Under Thorpe he carried out work on the oxides of phosphorus, which led to the recognition of the cause of phossy jaw, and to legislation which removed the danger of this disease from those who work in match factories.

Some of his best-known work, described in a book which has become a classic, was on the external forms of crystals. This was undertaken in part to investigate the extent to which the then relatively new law of isomerism was true, but it also led him to the preparation of many new crystals in remarkable perfection of form, established the extent to which the angles of crystals are in fact invariable, and provided an immense amount of valuable data.

In 1895 Tutton left South Kensington to become an Inspector of Technical Schools under the Board of Education, though his researches were continued in his private laboratory. During the time that he spent in London in this capacity, he designed for the Board of Trade a comparator on the interferometer principle, intended for the comparison of the standard yard with its copies. This instrument was but one of a number in which he used similar methods, others being an instrument for the determination of the distortion of crystals under stress, and his elasmometer and dilatometer, both for work on crystals. The comparator itself recorded in units of one eight-millionth of an inch, and Tutton intended to carry out a measurement of the wave-length of certain lines of H and Ne with it, but he was prevented from doing so by his transfer to Plymouth. After his retirement he returned to this subject and, using the rulings prepared for him by Grayson twenty years before, he completed the determination, which involved the actual counting of the number of wave-lengths in $\frac{1}{16}$ in. The result, computed in terms of cadmium radiation, agreed with that obtained by Sears and Barrell at the National Physical Laboratory to within one part in 1,400,000.

Tutton was a great lover of mountains and a member of the Alpine Club, and had visited the Rockies and Mount Kenya in East Africa. He was a member of the International Commission on Snow, and in this connexion his profound knowledge of glaciers was of great value.

REVIEWS OF BOOKS

Textbook of Heat, by R. W. STEWART, D.Sc. and Prof. JOHN SATTERLY; revised by C. T. ARCHER, M.Sc. Pp. vii + 410. (Second edition, University Tutorial Press, 1939.) *7s. 6d.*

The volume under review is intended for first-year and second-year university students, and in it one finds a clear exposition of a subject which, from the amount of historical matter included, one might be led to the conclusion that there has been but little advance during the last 30 years or so. Looking through the volume one cannot but feel that the time is ripe for a drastic pruning of the material provided for the digestion of first-year and second-year students. Too much space is given to Joly's steam calorimeter, which is now only of historic interest and has been little used during the past 50 years. To imply that Forbes's method is the right one for investigating the absolute thermal conductivity in the case of a metal is to give the student an incorrect conception of modern technique in this branch of the subject. Incidentally it might be noted that Forbes described his method 87 years ago. A careful search through the *Annual Reports on Progress in Physics* issued by the Physical Society would have helped to modernize the outlook.

There is, however, some additional matter in this new edition. For example, under gas thermometry one finds a description of the constant-volume gas thermometer of the International Bureau of Weights and Measures, Paris, although it comes as a slight shock to find that this was first described over half a century ago. Callendar's constant-pressure thermometer finds a place, but no mention is made of the very simple forms of gas thermometers used nowadays in research at very low temperatures.

The concluding paragraph of the section dealing with the gas thermometer states that the standard scale of temperature adopted for international use is "the Centigrade scale of the constant-volume hydrogen thermometer...." While it is true that the international scale was based on experimental work with the hydrogen gas thermometer, the definition of the international temperature scale nowadays contains no mention of the hydrogen thermometer. It is the thermodynamic centigrade scale which is recognized as the fundamental scale, and the experimental difficulties incident to the practical realization of the thermodynamic scale have made it expedient to adopt for international use a practical scale, designated as the "international temperature scale". This scale conforms with the thermodynamic scale as closely as is possible with present knowledge. The international temperature scale is based upon a number of fixed and reproducible equilibrium temperatures to which numerical values are assigned, and upon the indications of interpolation instruments, calibrated according to a specified procedure at the fixed temperatures. Thus, from the ice point to 660° C. the platinum resistance thermometer is used as the interpolation instrument.

In the chapter on specific heats, atomic heats are mentioned but the vast accumulation of theoretical and experimental work in this subject during the past 30 years is ignored. Possibly the explanation of the state of affairs is that the examiners responsible for the papers set to first-year and second-year students are not sufficiently interested in the subject of heat to set questions requiring up-to-date knowledge of the subject. So long as these conditions prevail there is little encouragement to authors to write modern books. But within these limitations the volume under review serves its purpose admirably.

E. G.

A Text Book of Heat, by H. S. ALLEN, F.R.S. and R. S. MAXWELL. Pt. I. Pp. v+525. (London: Macmillan and Co., Ltd., 1939.) 10*s*.

This is the first part of a two-volume work. It is intended for students preparing for Higher School Certificate or intermediate examinations in physics or for a university scholarship. The work is mainly descriptive and experimental and the mathematical treatment has been kept as simple as possible, although the notation of the calculus is introduced and explained. The development of the science of heat is traced from the speculations of the early philosophers to the more abstruse ideas connected with the quantum theory.

The volume represents a definite attempt to bring the treatment of the subject dealt with up to date. One finds for example a section on commercial applications of solids and liquids at low temperatures giving interesting information as to solid carbon dioxide and the separation of gases.

The treatment of thermal conductivity is good. One would, however, have liked to see an amplified account of the Lorenz law. It is stated that the law is satisfied at ordinary temperatures for pure metals, but it should be noted that there are exceptions to this law, particularly among the poorly conducting metals such as pure bismuth, antimony, and mercury, while some good conducting alloys obey the law.

In the chapter on latent-heat calorimetry one finds an account of Dewar's liquid-oxygen and liquid-hydrogen calorimeters by means of which he showed that "the atomic heats were periodic functions of the atomic weight and that the curve resembled, generally, the well-known Meyer atomic volume curve for the solid state". In connexion with the measurement of low temperatures reference might be made to the pentane thermometer which is very useful in the range from room-temperature down to the temperature of liquid oxygen. A minor point for criticism is the drawing of a sulphur boiling-point apparatus, figure 18, where the coil appears to project beyond the top of the sulphur bath.

An interesting feature is the biographical notes on distinguished workers in the subject. The quotation ascribed to Maxwell concerning Helmholtz will be new to many. It was he who first "effected a passage over that untrodden wild between acoustics and music—that Serbonian bog where whole armies of scientific musicians and musical men of science have sunk without filling it up".

The authors of the volume are to be warmly congratulated on their industry, which has resulted in the production of a book marking a distinct advance in the treatment of the subject. We shall await the appearance of the second part with keen interest. E. G.

A Text Book on Light, by A. W. BARTON. Pp. x+426. (London: Longmans, Green and Co., Ltd., 1939.) 8*s*.

According to the notice on the dust cover: "This book is an attempt to prepare the reader for a changing world by presenting the subject matter in its proper setting, by making him realize that scientific ideas are in a continual state of flux, developing out of well-established facts, growing and being modified as the theory itself suggests, and leading to the establishment of an ever-widening range of facts." As may be expected in consequence, the author eschews dogmatism and dwells more on the philosophical and educational aspects of his subject than is usual in works of similar scope.

In the part of the book devoted to geometrical optics, we note with pleasure that the author is careful to explain the application to finite objects of formulae proved for point objects, that he uses a symmetrical notation in connexion with refraction at a surface, and that he considers the formation of images by convergent incident light. Moreover he follows the Physical Society's *Report* in making use of the sign convention in which

distances measured from a refracting surface in the same direction as the initial direction of the light are counted positive; but it is noted with regret that he does not quote the useful definition of power which the *Report* contains. Modern photometry, illumination, and the determination of the velocity of light are dealt with in a manner which calls for commendation (but did Fizeau when performing his experiments use a concave mirror at the distant station?); and the subjects of wave motion, interferences, diffraction, polarization, and radiation are studied in some detail with emphasis on the evidence which they afford concerning theories of light. The treatment is straightforward, and, while no attempt is made to view the physics through a haze of symbols, the mathematical development is adequate.

Problems appended to each chapter will assist the Higher-Certificate, scholarship and pass-degree students to whom the book is addressed, and we predict that the volume will attain a popularity similar to that enjoyed by the author's *Heat*. E. J. I.

Electroacoustics, by E. MEYER. Pp. v + 117. (London: G. Bell and Sons, 1939.) 10*s*.

To those who were present at Prof. Meyer's series of lectures on electroacoustics at the Institute of Electrical Engineers in 1937 this book needs no introduction. These lectures are here published in a very readable form and are supplemented by many informative illustrations and a number of references. In this short book a wide field is covered, the subjects dealt with including the properties of sound waves, acoustical terms, supersonic sound, sound-analysis and intensity-measurement, microphones and loudspeakers, gramophone and magnetic recording, electrical music, room acoustics and sound-absorbents. Those engaged in research and in the various branches of industry in which electroacoustics are being used to an ever-increasing extent will find Prof. Meyer's book a useful summary of recent progress, particularly of that made by continental workers. Students will find it a convenient means of gaining an insight into an important and interesting branch of applied physics. K. M. C.

Terrestrial Magnetism and Electricity. Edited by J. A. FLEMING. Pp. xii + 794. (New York and London: McGraw Hill, 1939.) 52*s*. 6*d*.

This book is volume 8, the latest and largest, of a series of works included under the general title "Physics of the Earth", which have been prepared under a scheme formulated in 1926 by the United States National Research Council. The purpose of the series was "to give to the reader, presumably a scientist but not a specialist in the subject, an idea of its present status together with a forward-looking summary of its outstanding problems": hence this volume is "not a text-book, but a reference book which may stimulate the interest of a larger group of investigators in terrestrial magnetism and electricity".

One third of the book is devoted to terrestrial magnetism: the whole subject is briefly reviewed by Fleming, who also, with Johnston and McComb, deals with magnetic instruments and observations; Heiland writes on magnetic prospecting, McNish on the causes of the earth's magnetism and its changes, and Bartels on some problems of the subject.

The daily variations and irregular disturbance changes of the earth's magnetism largely originate in the upper atmosphere, particularly in the ionosphere, and the disturbances are closely associated with aurorae. Hence it is appropriate that more than a quarter of the book should deal with these subjects, in chapters by Berkner on the radio exploration of the ionosphere, by Hulburt on the upper atmosphere, and by Vegard on the aurora.

The rest of the general text is concerned with terrestrial electricity; Rooney contributes a chapter on earth currents, which offer many difficult problems of measurement and interpretation, and are connected with the magnetic changes. The other three electrical

chapters relate to the lower atmosphere; there is a general article on atmospheric electricity by Gish, one by Torreson on instruments and observations, and one by Schonland on thunderclouds, shower clouds and their electrical effects. These four chapters include nearly a quarter of the book.

The remaining hundred pages cover the last and longest chapter, by Harradon, on Bibliographical Notes and Selected References. The Preface rightly says that this chapter forms a most valuable research tool.

The work as a whole is an important and authoritative production, which seems certain to contribute materially to the advancement of its subject by making available to physicists and applied mathematicians, to geologists, engineers and others, the present state and the fascinating and difficult problems of this branch of earth-science. There is of course some over-lapping in the contributions by different authors, but this is at least in part advantageous in presenting the conflicting opinions which prevail on some aspects of earth magnetism.

S. C.

Theory and Design of Valve Oscillators, by H. A. THOMAS, D.Sc., M.I.E.E. Pp. xvii + 270. (London: Chapman and Hall, 1939.) 18*s*. net.

The technician often requires information on a particular subject more detailed and more up-to-date than can be found in the ordinary textbook. This book forms one of a series of monographs on electrical engineering designed, under the editorship of H. P. Young, to fulfil this need. It deals in greater detail than any previous book published in this country with the general principles of valve oscillators, the various types of oscillator, their efficiency and their stability in respect of frequency. There is a short chapter summarizing the different methods of adjusting the electrical circuit to increase the frequency stability; and the last chapter gives a useful account of mechanical methods of frequency monitoring. About one third of the book, drawn largely from the author's own works, is devoted to an analysis of the instabilities of inductance coils and condensers and describes some methods of overcoming them.

It is perhaps inevitable that a book written by a technical expert should suffer from a lack of balance. The latter part of this book could have been condensed with advantage, and there is throughout an unnecessarily detailed description of individual experiments and a lack of conciseness in style. On the other hand it is surprising that a book with the present title should not contain at least a brief description of a valve and some introductory explanations of the electrical components used and the technique of assembling them in a valve circuit. The author states in his preface that valve oscillators are used for numerous and varied purposes, but there is little mention of these in the text and no description of the oscillators suitable for them. The relaxation oscillator, for instance, receives only a general description which is of little use to the technician wishing to design an oscillator of this type. These omissions restrict the usefulness of the book, which nevertheless contains a good general description of valve oscillators, and information concerning coils and condensers which should be helpful to anyone designing these components.

The statement on p. 242 that the degree of stabilization of frequency with a valve master oscillator is not quite so high as that given by a quartz-controlled oscillator might in some cases be misleading; for it should be remembered that if the best models of the two types are compared the quartz oscillator is of the order of a thousand times better than the valve oscillator.

The book is well printed and only a few minor misprints were noticed.

L. E.

The Radio and Telecommunications Engineers' Design Manual, by R. E. BLAKEY, D.Sc. Pp. viii+142. (London: Sir Isaac Pitman and Sons, 1939.) 15*s*.

This book has been written primarily to assist the radio engineer in the design and construction of apparatus for use in the factory test room. While the author's plea that such a book should be non-academic in character is reasonable, it is hardly an excuse for presenting a volume which contains so many inaccuracies. These errors are not confined to the text and the mathematics, but extend to the circuit diagrams and are of such a nature as to make the reader suspicious of the book as a whole. Further defects in the presentation of the material, such as the occasional use of undefined symbols and the absence of an index, lead to the view that, in its present form, the book will not succeed in its object. This conclusion is highly unfortunate, as undoubtedly the subject matter might be arranged in the form of a valuable book on which the radio engineer could rely for practical information.

H. G. H.

Elektrische Höchstspannungen, by A. BOUWERS. Pp. ix+333. (Berlin: Julius Springer, 1939.) RM. 29.40, Bound RM. 31.20.

Such a book, from one who has so long been concerned with the technique of high-voltage production as Dr Bouwers of Eindhoven, would be certain, in normal times, to attain a wide publicity amongst physicists in this country. The choice, however, of a publisher in Berlin will unfortunately, from our point of view, render the book difficult to obtain here.

The book starts with a discussion of the various methods of high-voltage production, which discussion is so comprehensive as to include not only the methods available in the laboratory but nature's own contribution in the form of the lightning discharge. Also included is a consideration of the resonance methods for accelerating particles to high equivalent voltages by means of relatively low accelerating voltages. There follows a description of the electric fields caused by electrodes of varying geometrical configuration, and the methods available for mapping out such fields. The next section of the book deals with the properties of insulating materials, a wealth of technical information and data being given with regard to the electrical breakdown through the substance, and that across the surfaces, of these insulating media. The concluding sections of the book deal with the more specific problems which have to be met in the construction of high-voltage equipment. Amongst the problems discussed are the most suitable shape of insulator for use at high voltage and the construction of high-voltage condensers, transformers, resistances, rectifiers and switches. The methods available for the measurement of high voltages are given in detail together with much useful data. Finally the last section of the book deals with the "Use of high voltages", just in case the reader should have imagined that the attainment of high voltages was an end in itself. Here is to be found a discussion of the use of high voltages in nuclear physics, of the yield of neutrons ($^{7}_{3}\mathrm{Li}+^{2}_{1}\mathrm{H}$) at different voltages, and of the design of discharge tubes both for positive ions and for electrons. The book concludes by giving some good advice on the precautions which should be adopted in working with high voltages and also on the protective measures which should be taken in working with x rays or γ rays or with β rays or neutrons. Here again useful absorption data are given.

A feature of this excellent book is the beautifully produced diagrams, graphs, and photographs of apparatus. The bibliography consists of 324 references, most of which are to papers published within the last ten years.

W. B. M.

An Introduction to Crystal Chemistry, by R. C. EVANS, M.A., Ph.D., B.Sc. Pp. xi + 388. (Cambridge University Press, 1939.) 18*s*. net.

The object of this book, according to the preface, is to survey critically the broad field to which x-ray methods have been applied and to codify some of the principles of crystal architecture. This object is admirably achieved.

The first part is devoted to a consideration of the interatomic binding forces which come into play in a crystal, and to a discussion of lattice theory. In the second part a systematic treatment of crystal chemistry is given, the substances dealt with being classified in accordance with the types of interatomic forces concerned in their structures. Metals, alloy systems, organic and inorganic compounds, and liquids and gases are considered critically and in detail, their structures being correlated with their physical and chemical properties. Care is taken, however, that the main arguments are not obscured by structural data inserted for their own sake. Technical applications, such as chemical analysis by means of x rays, the determination of purity and grain-size, and the detection of strain, are omitted; such applications being expressly stated by the author to be outside the scope of the book.

A very useful feature is the profusion of summarizing tables from which the main results of the various sections may be seen at a glance. An extensive bibliography and a ong list of references add value to an already valuable book.

J. T.

Complex Variable and Operational Calculus, by N. W. MCLACHLAN. Pp. xii + 355. (Cambridge University Press, 1939.) 25*s*. net.

Dr McLachlan's work, particularly on loudspeakers, led him first into the remote intricacies of Bessel functions and on his return, like a good traveller, he published a most interesting account of the country. Now we have a book inspired by the subject of operational calculus, and bearing the title shown above, though the author is careful to stress that the method which he gives is not an operational one. The first chapter, that on complex variables, is introductory to the main theme of the book, and if it fails to attain even that minimum standard of rigour which is usually expected, no great harm is done, for no reader could tackle the main subject with no more preparation than is given there. It is followed by chapters on Cauchy's theorem, the calculus of residues and various other matters falling within the ordinary theory of functions.

The interest of the book lies in the second part, the theory of operational calculus. Dr McLachlan bases his theory on the Mellin inversion theorem in the form:

If $$\phi(p) = p\int_0^\infty e^{-pt} f(t)\, dt$$

then $$f(t) = \frac{1}{2\pi i}\int e^{pt}\phi(p)\, dp/p$$

where the second integral is taken round the Bromwich contour, a line parallel to the axis of y at such a distance as to leave all singularities of the integrand on its left. Then $\phi(p)$ is called the operational form of $f(t)$, and the contour integral is the *interpretation* of $\phi(p)$, although p is a parameter and not an operator. Thus if the operational form of a function is required, it is deduced from the formula

$$\phi(p) = p\int_0^\infty e^{-pt} f(t)\, dt.$$

It is interesting to see this method at work on a simple problem, such as that (given on p. 143 of the book) of solving the equation $y''+a^2y=Ce^{-bt}$ subject to the initial conditions $y=y'=0$.

The equation is multiplied by e^{-pt}, integrated from 0 to ∞, and multiplied by p, to give the result

$$p\int_0^\infty e^{-pt}\,dy'+a^2p\int_0^\infty e^{-pt}\,y\,dt=cp/(p+b).$$

Integrating the first term by parts brings the equation to the form

$$(p^2+a^2)\,p\int_0^\infty e^{-pt}\,y\,dt=cp/(p+b)$$

which is at once reducible to the standard form, and on inversion yields

$$y=\frac{c}{2\pi i}\int e^{zt}\,dz/(z+b)\,(z^2+a^2).$$

This is evaluated by means of its residues at the three poles, giving the usual solution,

$$y=\frac{c}{(a^2+b^2)}\left(e^{-bt}-\cos at+\frac{b}{a}\sin at\right).$$

Set out in this way, the method seems cumbersome and indeed for simpler problems it probably involves more steps than a frank appeal to the shorthand operational methods. But it has the advantage of keeping the mathematical aspect present throughout, and would not lend itself to the errors which have been made when pure operational calculus has been used.

Space is lacking to give a full account of the many investigations crowded into the book—a third of it is devoted to physical applications—but the original contributions are many, and there is no doubt that patient study of it would enable any reader possessing the necessary preliminary mathematical equipment to master all the theory necessary to enable him to investigate even difficult problems which are amenable to treatment by this method.

There is a full bibliography, and in an appendix are given the interpretations of many of the commonest operational forms.

J. H. A.

Higher Mathematics (*with Applications to Science and Engineering*), by R. S. Burington and C. C. Torrance. Pp. xiii+844. (New York and London: McGraw-Hill Publishing Company, 1939.) 30*s*. net.

The preface tells us that this volume is "an outgrowth of a series of courses in advanced calculus and related subjects, given by the authors at the Case School of Applied Science, and designed primarily to meet the growing needs of students interested in the applications of mathematics to physics and engineering". Although the physical interpretation of the mathematics is appropriately stressed, the presentation is sufficiently rigorous to make the book suitable for those whose interests are principally mathematical.

The ground covered is very wide indeed. There are altogether nine chapters, the first three of which, occupying rather more than half the space, deal on the usual lines with the differential and integral calculus and ordinary differential equations. Methods for the numerical solution of differential equations are not neglected, and the properties of systems of equations are generously illustrated by excerpts from electrical circuit theory. There is a brief reference to the generalized Lamé equation, with a fairly detailed treatment of the cases of the Legendre and Bessel equations. We next come to a chapter on infinite

sequences and series, including an extensive treatment of Fourier series, though very little on the Fourier integral, and then to one on the functions of a complex variable, which contains not only most of the usual theorems, but also an introduction to conformal representation and some useful notes on elliptic functions. A good chapter on algebra and vector analysis follows, where in addition to sections dealing with matrices, determinants and vectors, the authors have found room for a very readable introduction to the differential geometry of curves and surfaces, and a short account of the tensor calculus. The remaining topics include partial differential equations, the calculus of variations, and finally a brief introduction to the theory of the real variable. Among notable absentees, integral equations and asymptotic expansions may be mentioned. There is admittedly a reference to the latter in the index, but on turning up the page concerned we find that it refers only to the development of functions in Laurent series, and contains no reference to asymptotic expansions, which, moreover, do not appear to be mentioned anywhere else in the book. In the reviewer's opinion the balance of the book, already good, could be made even better if the first two chapters were slightly shortened to make room for brief treatments of a few additional topics such as those mentioned above. This is put forward purely as a suggestion for future editions, for the book deals already with such a wide diversity of subjects that any criticism on the ground of omissions would be ungracious.

There are one or two departures from standard notation which call for comment. There seems to be little point in introducing the integral symbol $I_a f(x)$ in place of the usual $\int f(x)\,dx$, only to discard it after a few pages in favour of the standard, or as the authors call it the "historical", form. Again, it is not easy to resign oneself to such expressions as ctn, csc, ctnh and csch in place of the usual cot, cosec, coth and cosech; the customary terms at least have the merit of being reasonably pronounceable. These are very minor criticisms, however, and the book can be confidently recommended to students and others who are looking out for a reasonably concise general textbook on applicable mathematics, which is at the same time reliable and interesting. An attractive feature is the large number of examples, which incidentally include many theorems for which there is not space in the text. There is also a good bibliography which should assist those who wish to pursue further the topics initiated in the book.

R. S. D.

Problems in Mechanics, edited by G. B. Karelitz, J. Ormondroyd and J. M. Garrelts. Pp. ix+271. (New York: The Macmillan Company, 1939.) 11*s*.

This excellent book contains nearly 800 well chosen and graded examples in statics and dynamics, the majority of which are illustrated by clear diagrams. In nearly every case the answer is given, and about 10 per cent of the problems are provided with brief solutions "to suggest to students a method of attack which they can follow to advantage in handling the rest of the problems". In their preface the authors state: "The exposition of the principles and theorems of mechanics is of little practical value to the student unless he is constantly exercised in their application to actual problems. Only by this means can mechanics become a working tool for the future engineer." With this statement every teacher will agree. This book provides the means as most of the examples are based on practical problems. A brief summary of the theorems required enhances the value of the book, which should make a wide appeal to all teachers and students of mechanics whether engineers or not.

J. G. D.

Aircraft Design: Vol. II, Aerostructures, by C. H. Latimer Needham. Pp. xiii+308. (London: Chapman and Hall, Ltd., 1939.) 16*s*.

This book is the second volume of a work on aircraft design and—though its title may suggest a wider field—deals solely with the design of aeroplanes from the structural

standpoint. After some preliminary general chapters on the arrangement of aeroplane structures and the materials used, a series of chapters are devoted to brief discussions on wings, fuselages, tail units, undercarriages, and controls. Short chapters are included on structural strength calculations, airscrew and engine properties, some characteristics of seaplanes and flying boats, and metal construction.

The book is characterized by a clarity of description and a simplicity of exposition regarding design problems that will no doubt help it to fulfil one of its intended functions as "a guide for private constructors". But in attempting to cover the whole field of structural design in the space of 300 pages the author has set himself a difficult task. It is easy for such a work to become a confusion of general principles on one hand and design details on the other—a mixture troublesome to an engineer and still more displeasing to a physicist. And though he tries to deal "with the general principles of aircraft design", the author has too often failed to express structural design in terms of physical principles as generally understood. Thus the discussion of the loading experienced by an aeroplane is too much like a good summary of official airworthiness regulations and there is too little remark on the functions of the various structural components of an aeroplane.

This is a defect from which other textbooks on the same subject suffer; and like them, also, this book is largely written, though not ostensibly and perhaps to a rather less degree, around the wooden biplane of the past. This is a great pity, for the young student, seeing around him the metal-clad monoplanes of the Royal Air Force of to-day and eager to learn how they are constructed, must be sadly disappointed by many of the examples of this textbook, and must tend to lose interest in what is really a most absorbing subject to mathematicians, engineers, and physicists alike.

A. G. P.

The Scientific Journal of the Royal College of Science. Vol. IX. Pp. 137. (London: Edward Arnold and Co.) 7*s*. 6*d*.

The three societies—Chemical, Natural History, and Mathematical and Physical—of the Royal College of Science combine to publish a journal, of which this is the ninth annual volume. This contains only a selection from the papers and addresses given to the societies, and most of those printed consist of surveys of some interesting fields of knowledge. Among the chemical subjects are two showing how chemistry may make its contribution to easing the lot of mankind; the one is on the dusts responsible for silicosis and the other on carcinogenic (cancer-causing) agents. The Natural History section contains accounts of two scientific expeditions, and the Physical and Mathematical Society prints a lecture by Dr Mann on the cyclotron, one by Dr A. H. Davis on the measurement of sound and noise, and two particularly interesting ones in which Prof. Dingle and Prof. Levy set out their views on what is generally called the philosophy of science. These are such admirable summaries of their two points of view that they deserve to be known to wider audiences. Among the other lecturers during the year were Profs. G. P. Thomson, E. K. Rideal, G. I. Finch, A. C. Egerton and Dr C. G. Darwin.

J. H. A.

Modern Science, by HYMAN LEVY. Pp. x+736. (London: Hamish Hamilton, 1939.) 21*s*.

The task which Prof. Levy has undertaken, that of giving a comprehensive survey of physical science in the world of to-day for the ordinary intelligent person, is one which most of those concerned with physics would agree to be well worth while. However much they might disagree with some of the particular points of view which the author expresses, they could not but have the highest admiration for the easy mastery with which he handles his material and for the energy and verve with which he has carried out his elaborate programme.

The essential idea underlying the presentation is that science is one of the forms in which social energy manifests itself, and that it cannot be considered, without serious misrepresentation, apart from its social background and its effects on social practice. It is the insistence on this point of view, rather than the mere acceptance of it, which gives the book its special character. Much of the scientific ground covered, however, has been well trodden in some of the excellent books already available on science for the general reader, and it is unfortunate that the author should appear to ignore their existence. Surely one of the first duties of the socially minded writer on science is to indicate the relation of his own presentation to that of others.

The first part of the book treats historically some of the main phases in the development of science, and traces the relation between science and its social background through the ages. At the present time the author sees an inversion of science, with capitalism inevitably involved in war and mobilizing science for that purpose. The scientist, "at once the product and the servant of a dying culture... holds in his hands the key to the making of a new".

The bulk of the book, parts II–VII, occupying some 600 pages, presents, with a wealth of illustration, some of the leading themes of physics. A somewhat discursive survey of scientific method and procedure, in the course of which the forms and movements of energy are considered at length, is followed by a discussion of "The universe of symbols". This outlines the development of arithmetic, algebra and the calculus, and shows clearly the point of many of the apparently abstract and recondite problems of mathematics. The absolute machine is then considered, and the Newtonian laws of motion are subjected to detailed scrutiny. As an exemplification of the practical bearing of classical physics, a long chapter is devoted to an account of the basic principles of aeronautical science. Part V, entitled "The perfection of shape", tells "the sad story of Euclidean geometry" and discusses its extension. The present answer to the question, "What is the Universe?", as far as a scientific answer can be given, is outlined in part VI. The stars, the solar system and the earth are surveyed in turn. The bearing of the theory of relativity is indicated. The complementary field view and particle view of interaction processes are treated, and finally the pattern in detail is described, stress being laid, in the account of atomic and electronic phenomena, on the dominance of statistical laws. To illustrate the bearing of modern science on practice, an excellent account is given of the elaborate interplay of theory, experiment, and diverse technique involved in the development of modern methods of illumination.

In an epilogue, "Science in travail", the author stresses two points which he regards the survey as having indicated—the interrelation between science and social activity, and the growth of science through the resolution of internal contradiction by a higher synthesis. He also presents some of the characteristics of the older mechanistic and the newer dialectical method in a table of nineteen contrasting statements, which appears to give on one side the view of some idealized mechanist, and on the other those of the author. Why the newer method is called dialectical is left obscure, nor is it clear why such a statement as "The speed of light is a statistical constant" is characteristic of it. Analysis of scientific method in a sense which others might regard as more fundamental is, however, foreign to the author's purpose.

Although few would deny that the statement "The history of science and the growth of its ideas are intimately bound up with the history of social needs" is in accordance with a wide range of historical evidence, yet in its implications it seems to leave out of account the fact that there are scientific ideas, including some to which high value is attached by scientists, that seem to have no bearing on social needs. It is striking that the author is sometimes at his happiest when expounding ideas which seem to be socially useless. It is this which may possibly leave readers with a feeling of inconclusiveness about what purports to be a central argument of the book, quite apart from whether or not they are

in sympathy with the author's general social and political views. This, however, does not detract from the general excellence of handling of the scientific themes themselves. The ordinary intelligent person has a journey full of interest before him when he tackles this book, and the occasional difficult passages should add a stimulus. It is to be hoped that the book will be read by many who are not professionally concerned with physical science, as well as by those who are. The many diagrams and sketches, due to Joan Samuell, boldly executed with significant economy of detail, add much to the attractiveness of an admirably produced volume.

E. C. S.

German-English Science Dictionary, by LOUIS DE VRIES. Pp. x+473. (London: McGraw-Hill Publishing Co., 1939.) 19*s.* 9*d.*

The plan on which this dictionary was compiled sounds ideal; the professor of modern languages prepared it, with the active co-operation of the staffs of the various scientific departments of the Iowa State College, who pointed out to him, in particular, the standard works in each subject which should be combed for the necessary technical vocabulary. It is nevertheless open to criticism in detail. The lexicographer prides himself on the fact that it contains 48000 entries, but he might have produced a cheaper and smaller book of equal utility by reducing the number of words which are identical, or nearly so, in English and German. The writer who aspires to publish in German may need to know that *direkt* is the German for *direct*, but the scientist who is merely hoping to read German technical literature will surely not need to look this up in a dictionary. Nearly every page contains similar examples, and the following are taken at random: Kodex, kohäsiv, fiktiv, fibrös, Dioxyd, Analogie, Aconit, Methode, Artikel, explodieren, Anode, and extensiv.

On the credit side is the large number of biological terms, each accurately defined, generally with the technical (Latin) term. Indeed, the book appears to have the biological rather than the physical sciences in mind, and it often misses the specifically mathematical meanings of words. Thus under *Ableitung* we find "drainage, derivation, deduction" but not "derivative, differential coefficient", and under *Ausgleichen* we have "to equalize, adjust, compensate, balance, level, settle, arrange", but not "to equate". Incidentally, is it by coincidence or due to derivation from a common source, that another well-known science dictionary gives for *Ableitung* and *Ausgleichen* "drainage; derivation; derivative; deduction, channel; turning aside", and "equalize, adjust; compensate; balance; level; settle", respectively?

Under *Schalter* there are the entries "ruler, manager, wicket, window", though to the physicist the commonest meaning is "switch", and in fact the other dictionary just mentioned gives "wicket, window; ruler, manager; (*Elec.*) switch", and incidentally by its groupings makes clearer what sort of wicket and what sort of ruler are intended.

Among less technical words, the dictionary gives help in many useful ways, but there is an unfortunate misprint whereby *halber* in its sense of "on behalf of" fails to appear. *Mittelmeer* and *Bodensee* are both given, but not *Südsee* or *Ostsee*.

The book is beautifully produced, and the choice of type is such that words stand out well, yet not in such a way as to weary the eye. For those who already have an engineering dictionary, or whose interest is mainly in zoology or botany, the book is probably of extreme value, but it hardly fulfils the needs of the ordinary physicist, and it is to be hoped that another edition will soon appear in which he will be better catered for.

J. H. A.

Under the general heading *Actualités Scientifiques et Industrielles* we have received the monographs listed below. Each is written by an authority on his subject and the treatment is, in general, concise and clear. The publishers are Hermann and Co., 6 Rue de la Sorbonne, 6, Paris.

653. Pierre Fleury. *Mesure des Temps, Vitesses, Débits.* 30 fr.

654. M. Matricon. *Application de la Méthode du Champ Self-Consistent aux Noyaux Atomiques.* 25 fr.

655. Philip A. Leighton. *Determination of the Mechanism of Photochemical Reactions.* 18 fr.

656. Gerhard K. Rollefson. *The Photochemistry of the Halogens.* 20 fr.

658. Frédéric Bremer. *L'Activité Électrique de l'Écorce Cérébrale.* 15 fr.

662. Ernest Kahane and Jeanne Lévy. *Choline-Neurine.* 15 fr.

693. Pierre Brunet. *Étude Historique sur le Principe de la Moindre Action.* 30 fr.

703. Harry Willstaedt. *L'Analyse Chromatographique et ses Applications.* 30 fr.

704. Louis de Broglie. *Le Principe de Correspondance et les Interactions entre la Matière et le Rayonnement.* 10 fr.

709. Aloys Rey. *Études sur l'Écoute des Sources Sonores Éloignées.* 21 fr.

718. P. Debye, F. Simon, M. Wiersma, Sir C. V. Raman, M. Polanyi and B. van der Pol. *Physique Générale.* 21 fr.

719. N. Bohr, P. Scherrer, J. D. Cockcroft and W. Bothe. *Physique Nucléaire.* 11 fr.

720. J. Clay, P. M. S. Blackett and G. Lemaitre. *Rayons Cosmiques.* 10 fr.

729. Jean-Louis Destouches. *Les Électrons Lourds (Mésotons).* 21 fr.

731. Maurice Prost. *Optique, Électricité.* 21 fr.

INDEX

PAGE

Acoustic spectra obtained by the diffraction of light from sound films. . 244, 549
Alloys, The optical constants of copper-nickel 942
Amalgams, The electrical resistance of ferromagnetic 778
Amalgams, The electrical resistance of nickel 419
Andrade, E. N. da C.: The velocity-distribution in a liquid-into-liquid jet. Part 2: The plane jet 784
Anthracene, The diffraction of electrons by 479
Antimony fluoride (SbF), The band spectrum of 329
Appleton, E. V., R. Naismith and L. J. Ingram: The critical-frequency method of measuring upper-atmospheric ionization 81
Astbury, N. F. and L. H. Ford: The precision measurement of capacitance . 37
Atmosphere, The intermediate ions of the 1014
Atmospherics, The reception of broadcast signals subject to disturbance by . 972

Band-absorbed solar radiation, The atmospheric height distribution of . . 93
Band system of carbon monoselenide, A reinvestigation of the ultra-violet . . 989
Bands of germanium monoxide, Rotational analysis of the ultra-violet . . 62
Barkas, W. W.: Analysis of light scattered from a surface of low gloss into its specular and diffuse components 274
Barnard, G. P.: The spectral sensitivity of selenium rectifier photoelectric cells . 222
Barrow, R. F.: A reinvestigation of the ultraviolet band system of carbon monoselenide 989
Barrow, R. F.: Ultraviolet band systems of silicon monoselenide and monotelluride 267
Bates, L. F. and W. P. Fletcher: The electrical resistance of ferromagnetic amalgams 778
Bates, L. F. and J. H. Prentice: The electrical resistance of nickel amalgams . 419
Bates, L. F. and G. G. Taylor: Ferromagnetic compounds of chromium . . 33
Beevers, C. A.: A machine for the rapid summation of Fourier series . . 660
Bennett, G. A., *see* Irons, Eric J.
Berry, J. K., *see* Brentano, J. C. M.
Beta particles of xenon nuclei, The scattering of fast 355
Beta-ray spectra, The relation between range and energy for the upper limits of 19
Blackman, M., *see* Thomson, G. P.
Bor, J., A. Hobson and C. Wood: The application of a new photoelectric method to the determination of the optical constants of some pure metals . . 932
Bor, J., A. Hobson and C. Wood: The optical constants of copper-nickel alloys. 942
Bound, M. and D. A. Richards: A study of the atmospheric oxidation of metals and alloys at different temperatures by electron diffraction . . . 256
Brentano, J. C. M., Joan Honeyburne and J. K. Berry: Measurements of the intensity-distribution of the white x radiation reflected from a crystal; with a note on the dispersion of the atomic scattering factor of zinc near the K absorption edge 668
Brightness, The effect of adaptation and contrast on apparent 817
Brindley, G. W. and P. Ridley: An x-ray investigation of atomic vibrations in cadmium 73

PAGE

Brindley, G. W. and P. Ridley: An x-ray study of lattice-distortion in copper, nickel and rhodium 432
Brindley, G. W. and P. Ridley: The characteristic temperature of magnesium oxide 69
Bristow, J. R.: The conductivity of thin films of thallium on a pyrex glass surface 349
Broadcast signals subject to disturbance by atmospherics, The reception of . 972
Broadway, L. F. and A. F. Pearce: Emission of negative ions from oxide cathodes 335
Brown, D.: Acoustic spectra obtained by the diffraction of light from sound films 244, 549

Cabrera, B. and A. Duperier: Further results on the magnetism of chlorides of the palladium and platinum triads of elements 845
Cadmium, An x-ray investigation of atomic vibrations in 73
Campbell, Albert: Mutual and self inductors compensated for change of frequency 539
Campbell, Albert: Simple demonstration of the Peltier effect with approximate measurement 545
Capacitance, The precision measurement of 37
Carbon and graphite between 0° c. and 900° c., The variation with temperature of the electrical resistance of 147
Carbon and graphite at high temperatures, The thermal and electrical conductivities of 153
Carbon arc, CN and AlO bands in the spectrum of the 733
Carbon monoselenide, A reinvestigation of the ultraviolet band system of . . 989
Cathcart, Eliza B., *see* Emeleus, K. G.
Chapman, F. W.: Atmospheric disturbances due to thundercloud discharges. Part I 876
Chapman, S.: The atmospheric height distribution of band-absorbed solar radiation 93
Characteristic temperature of magnesium oxide, The 69
Charlesby, A., G. I. Finch and H. Wilman: The diffraction of electrons by anthracene 479
Chipman, R. A.: Ultra-high-frequency resonances in the positive-grid triode . 566
Chromium, Ferromagnetic compounds of 33
Chronograph, An accurate hard-valve counter 1028
Collier, L. J., W. S. Stiles and W. G. A. Taylor: The variation with temperature of the electrical resistance of carbon and graphite between 0° c. and 900° c. 147
Conductivities of carbon and graphite to high temperatures, The thermal and electrical 153
Conductivity of iron at high temperatures, Further measurements of the thermal and electrical 407
Conductivity of thin films of thallium on a pyrex glass surface, The 349
Constable, J. E. R., *see* Obituary notices
Constable, J. E. R.: Transmission of sound between neighbouring rooms in a brick building 53
Copper, nickel and rhodium, An x-ray study of lattice-distortion in 432
Cosmic-ray showers underground and at sea-level, Comparison of 585
Cosmology, Observation and theory in 529
Cosmology, Relativistic 465
Crank, J., D. R. Hartree, J. Ingham and R. W. Sloane: Distribution of potential in cylindrical thermionic valves 952
Critical-frequency method of measuring upper-atmospheric ionization, The . 81
Crystal rectification, The effect of cooling and magnetic fields on 449

PAGE

Cyclotron, A mechanical model illustrating the principle of the . . 810

Dental enamel by x-rays and the polarizing microscope, The examination of . 992
Diamonds by the method of mixtures, Measurement of the specific inductive capacity of 173
Dielectrics at centimetre wave-lengths, A new method of determining the properties of 689
Diffraction analysis of the relation between height and gain for radio waves of medium lengths, The experimental verification of the 805
Diffraction of light by supersonic waves, The influence of the angle of incidence of light on the 794
Discharge tube, Demonstration of a heavy-current hydrogen 551
Dispersion of a prism, The 803
Duperier, A., *see* Cabrera, B.

E region, Theoretical ionization curves for the 138
Eckersley, T. L. and G. Millington: The experimental verification of the diffraction analysis of the relation between height and gain for radio waves of medium lengths 805
Eckersley, T. L. and G. Millington: The limiting polarization of medium waves reflected from the ionosphere 110
Electrical conductivities of carbon and graphite at high temperatures, The thermal and 153
Electrical conductivity of iron at high temperatures, Further measurements of the thermal and 407
Electrification, A new method of creating 402
Electrification of powders by free fall, The 1010
Electroacoustics, Discussion on 359
Electron-diffraction, A study of the atmospheric oxidation of metals and alloys at different temperatures by 256
Electron-diffraction at high temperature, Apparatus for 237
Electron-diffraction, Theory of the width of rings formed by 425
Electronic semi-conductors, On the theory of 575
Electron lenses, The investigation of 296, 376
Electrons by anthracene, The diffraction of 479
Emeleus, K. G., R. H. Sloane and Eliza B. Cathcart: Excitation of the green auroral line of oxygen 978

Ferromagnetic compounds of chromium 33
Films of thallium on a pyrex glass surface, The conductivity of thin . . . 349
Finch, G. I., *see* Charlesby, A.
Fleming, J. A.: March 21, 1874. I. On the new contact theory of the galvanic cell 393
Fleming, Sir Ambrose: A new method of creating electrification . . . 402
Fleming, Sir Ambrose: Physics and the physicists of the eighteen-seventies . 383
Fletcher, W. P., *see* Bates, L. F.
Follett, D. H.: Comparison of cosmic-ray showers underground and at sea-level 585
Ford, L. H., *see* Astbury, N. F.
Fourier series, A machine for the rapid summation of 660

Galvanic cell (March 21, 1874). I. On the new contact theory of the . . . 393
Gamma-ray measurement of radium, The 905

PAGE

Garton, W. R. S.: Demonstration of a heavy-current hydrogen discharge tube . 551
Geiger-Muller counters, The extinction of discharges in 26
Germanium monoxide, Rotational analysis of the ultraviolet bands of . . 62
Goche, O. and H. Wilman: The structure of silver films 625
Graphite between 0° c. and 900° c., The variation with temperature of the electrical resistance of carbon and 147
Graphite at high temperatures, The thermal and electrical conductivities of carbon and 153
Guest, J. J.: Shear-stress systems and combined stress 652
Gupta, A. K. Sen: Rotational analysis of the ultraviolet bands of germanium monoxide 62
Gupta, R. L. Sen: The scattering of fast beta particles by xenon nuclei . . 355

Hackett, W., *see* Whitehead, S.
Hartree, D. R., *see* Crank, J.
Harvey, A., *see* Sutton, Miss M. E.
Hill, A. V.: The transformations of energy and the mechanical work of muscles 1
Hobson, A., *see* Bor, J.
Hogg, A. R.: The intermediate ions of the atmosphere 1014
Honeyburne, Joan, *see* Brentano, J. C. M.
Howell, H. G. and G. D. Rochester: The band spectrum of antimony fluoride (SbF) 329
Hutchinson, G. L., *see* Velasco, M.
Hydrogen on platinum and the removal of these gases by positive-ion bombardment, The adsorption of oxygen and 318

Inductive circuit. A falling-weight time switch; its standardization and application to the determination of the time constant of an 459
Inductors compensated for change of frequency, Mutual and self- 539
Ingham, J., *see* Crank, J.
Ingram, L. J., *see* Appleton, E. V.
Insulin, A note on the structure of 613
Ionization curves for the E region, Theoretical 138
Ionization, The critical-frequency method of measuring upper-atmospheric . 81
Ionosphere, The dispersion of wireless echoes from the 859
Ionosphere, The limiting polarization of medium waves reflected from the . 110
Ionospheric calculations, The effect of the Lorentz polarization term in . . 747
Ions from oxide cathodes, Emission of negative 335
Ions of the atmosphere, The intermediate 1014
Irons, Eric J. and G. A. Bennett: A falling-weight time switch; its standardization and application to the determination of the time constant of an inductive circuit 459

Jackson, R. and A. G. Quarrell: Apparatus for electron-diffraction at high temperatures 237
Jet, The velocity distribution in a liquid-into-liquid. Part 2: The plane jet . 784
Johnson, B. K.: An achromatic objective for use in ultra-violet microscopy . 1035

Klemperer, O. and W. D. Wright: The investigation of electron lenses . . 296
Krishnamurty, S. G., *see* Rao, A.
Krypton, Kr IV. The third spark spectrum of 772

PAGE

Langmuir, Irving and Dorothy Wrinch: A note on the structure of insulin . 613
Langmuir, Irving: The structure of proteins 592
Lead, The rate of viscous flow of 203
Lee, H. W.: Oblique refraction through a cylinder 768
Lenses, The investigation of electron 296
Light scattered from a surface of low gloss into its specular and diffuse components, Analysis of 274

MacLeod, Major-General M. N.: Some recent developments in British surveying instruments 710
Magnesium oxide, The characteristic temperature of 69
Magnetism of chlorides of the palladium and platinum triads of elements, Further results on the 845
Manometer, The thermal, a new device for recording low absolute pressures . 1003
May, A. Nunn: The extinction of discharges in Geiger-Muller counters . . 26
McVittie, G. C.: Observation and theory in cosmology 529
Mercer, E. H.: Spreading of paraffin oils containing fatty acids on aqueous solutions of calcium ions 561
Microscopy, An achromatic objective for use in ultraviolet 1035
Millington, G., *see* Eckersley, T. L.
Mond, Sir Robert, F.R.S., *see* Obituary notices
Moore, R. L.: The reception of broadcast signals subject to disturbance by atmospherics 972
Morris, G.: The electrification of powders by free fall 1010
Muscles, The transformations of energy and the mechanical work of . . . 1

Naismith, R., *see* Appleton, E. V.
Neumann, E. A.: The influence of the angle of incidence of light on the diffraction of light by supersonic waves 794
Nickel amalgams, The electrical resistance of 419
Nickel, copper and rhodium, An X-ray study of lattice-distortion in 432
Nicol, J.: A simple demonstration of the Volta contact E.M.F. 550
Nijboer, B. R. A.: On the theory of electronic semi-conductors 575

Oatley, C. W.: The adsorption of oxygen and hydrogen on platinum, and the removal of these gases by positive-ion bombardment 318
Obituary notices 553
Objective for use in ultra-violet microscopy, An achromatic 1035
Oddie, T. H.: The gamma-ray measurement of radium 905
Optical constants of copper-nickel alloys, The 942
Optical constants of some pure metals, The application of a new photoelectric method to the determination of the 932
Owen, D.: Note on the bar pendulum 456
Oxidation of metals and alloys at different temperatures by electron-diffraction, A study of the atmospheric 256
Oxygen and hydrogen on platinum and the removal of these gases by positive-ion bombardment, The adsorption of 318
Oxygen, Excitation of the green auroral line of 978

Palladium and platinum triads of elements, Further results on the magnetism of chlorides of the 845

PAGE

Paraffin oils containing fatty acids on aqueous solutions of calcium ions, The spreading of 561
Pearce, A. F., *see* Broadway, L. F.
Peltier effect with approximate measurement, Simple demonstration of the . 545
Pendulum, Note on the bar 456
Photoelectric cells, The spectral sensitivity of selenium rectifier . . . 222
Photoelectric method to the determination of the optical constants of some pure metals, The application of a new 932
Photographic films to X radiation at very low temperatures, The sensitivity of . 683
Photometric observer (2), The relative luminosity of radiation for the average . 757
Physics and the physicists of the eighteen-seventies 383
Piddington, J. H.: The origin of radio-wave reflections in the troposphere 129, 547
Pitt, F. H. G.: The effect of adaptation and contrast on apparent brightness . 817
Platinum, The adsorption of oxygen and hydrogen on, and the removal of these gases by positive-ion bombardment 318
Platinum thermocouple at the freezing points of gold, silver and antimony, The reproducibility of the 695
Platinum triads of elements, Further results on the magnetism of chlorides of the palladium and 845
Polarization of medium waves reflected from the ionosphere, The limiting . . 110
Polarization term in ionospheric calculations, The effect of the Lorentz . . 747
Powders by free fall, The electrification of 1010
Powell, R. W. and F. H. Schofield: The thermal and electrical conductivities of carbon and graphite at high temperatures 153
Powell, R. W.: Further measurements of the thermal and electrical conductivity of iron at high temperatures 407
Prentice, J. H., *see* Bates, L. F.
Preston, J. S.: The relative luminosity of radiation for the average photometric observer (2) 757
Prism, The dispersion of a 803
Proteins, The structure of 592

Quarrell, A. G., *see* Jackson, R.

Radio-wave reflections in the troposphere, The origin of 129, 547
Radio waves of medium lengths, The experimental verification of the diffraction analysis of the relation between height and gain for 805
Radium, The gamma-ray measurement of 905
Rao, A. Balankeswara and S. G. Krishnamurty: The third spark spectrum of krypton, Kr IV 772
Ratcliffe, J. A.: The effect of the Lorentz polarization term in ionospheric calculations 747
Rectification, The effect of cooling and magnetic fields on crystal 449
Reekie, James: The sensitivity of photographic films to X radiation at very low temperatures 683
Refraction through a cylinder, Oblique 768
Relativistic cosmology 405
Resistance of carbon and graphite between 0° C. and 900° C., The variation with temperature of the electrical 147
Resistance of ferromagnetic amalgams, The electrical 778
Resistance of nickel amalgams, The electrical 419
Reviews of books 191, 378, 557, 741, 895, 1049

PAGE

Rhodium, An x-ray study of lattice distortion in copper, nickel and . . . 432
Richards, D. A., *see* Bound, M.
Ridley, P., *see* Brindley, G. W.
Rochester, G. D., *see* Howell, H. G.

Schofield, F. H., *see* Powell, R. W.
Selenium rectifier photoelectric cells, The spectral sensitivity of . . . 222
Selincourt, M. de: The reproducibility of the platinum thermocouple at the freezing points of gold, silver and antimony 695
Shear-stress systems and combined stress 652
Sherbini, M. A. El and Y. L. Yousef: The effect of cooling and of magnetic fields on crystal rectification 449
Silicon monoselenide and monotelluride, Ultraviolet band systems of . . 267
Silver films, The structure of 625
Simons, A., *see* Vincent, R. S.
Sloane, R. H., *see* Emeleus, K. G.
Sloane, R. W., *see* Crank, J.
Solar radiation, The atmospheric height distribution of band-absorbed . . 93
Sound between neighbouring rooms in a brick building, Transmission of . . 53
Sound films, Acoustic spectra obtained by the diffraction of light from . 244, 549
Spectra, The relation between range and energy for the upper limits of beta-ray 19
Spectral sensitivity of selenium rectifier photoelectric cells, The . . . 222
Spectrum of antimony fluoride (SbF), The band 329
Spectrum of krypton, Kr IV, The third spark 772
Spreading of paraffin oils containing fatty acids on aqueous solutions of calcium ions, The 561
Stiles, W. S., *see* Collier, L. J.
Straker, T. W., *see* White, F. W. G.
Stress, Shear stress systems and combined 652
Supersonic waves, The influence of the angle of incidence of light on the diffraction of light by 794
Surveying instruments, Some recent developments in 710
Sutton, Miss M. E. and A. Harvey: The dispersion of a prism 803

Tawde, N. R. and S. A. Trivedi: CN and AlO bands in the study of the carbon arc 733
Taylor, G. G., *see* Bates, L. F.
Taylor, W. G. A., *see* Collier, L. J.
Temple, G.: Relativistic cosmology 465
Thallium on a pyrex glass surface, The conductivity of thin films of . . . 349
Thermal and electrical conductivities of carbon and graphite to high temperatures, The 153
Thermal and electrical conductivity of iron at high temperatures, Further measurements of the 407
Thermionic valves, Distribution of potential in cylindrical 952
Thermocouple at the freezing points of gold, silver and antimony, The reproducibility of the platinum 695
Thewlis, J.: The examination of dental enamel by x rays and the polarizing microscope 992
Thomson, G. P. and M. Blackman: Theory of the width of rings formed by electron-diffraction 425
Thundercloud discharges: Part I, Atmospheric disturbances due to 876

PAGE

Time-switch; its standardization and application to the determination of the time constant of an inductive circuit, A falling weight 459
Triode, Ultra-high-frequency resonances in the positive-grid 566
Trivedi, S. A., *see* Tawde, N. R.
Troposphere, The origin of radio-wave reflections in the 129, 547
Tyte, L. C.: The rate of viscous flow of metals. Part 2: Lead 203

Uffelmann, F. L.: An accurate hard-valve counter chronograph 1028
Ultraviolet band systems of silicon monoselenide and monotelluride 267
Ultraviolet bands of germanium monoxide, Rotational analysis of the . . . 62

Velasco, M. and G. L. Hutchinson: A new method of determining the properties of dielectrics at centimetre wave-lengths 689
Vibrations of free square plates. Part I: Normal vibrating modes 831
Vincent, R. S. and A. Simons: The thermal manometer, a new device for recording low absolute pressures 1003
Viscous flow of metals, The rate of. Part 2: Lead 203
Volta contact E.M.F., A simple demonstration of the 550

Waller, Mary D.: Vibrations of free square plates. Part I: Normal vibrating modes 831
Ward, F. A. B.: A mechanical model illustrating the principle of the cyclotron . 810
White, F. W. G.: The dispersion of wireless echoes from the ionosphere . . 859
White, F. W. G. and T. W. Straker: The diurnal variation of absorption of wireless waves 865
Whitehead, S. and W. Hackett: Measurement of the specific inductive capacity of diamonds by the method of mixtures 173
Widdowson, E. E.: The relation between range and energy for upper limits of beta-ray spectra 19
Wilkes, M. V.: Theoretical ionization curves for the E region 138
Wilman, H., *see* Charlesby, A.
Wilman, H., *see* Goche, O.
Wireless echoes from the ionosphere, The dispersion of 859
Wireless waves, The diurnal variation of absorption of 865
Wood, C., *see* Bor, J.
Wright, W. D., *see* Klemperer, O.
Wrinch, Dorothy, *see* Langmuir, Irving

Xenon nuclei, The scattering of fast beta particles by 355
x radiation at very low temperatures, The sensitivity of photographic films to . 683
x radiation reflected from a crystal, Measurements of the intensity-distribution of the white; with a note on the dispersion of the atomic scattering factor of zinc near the K absorption edge 668
x-ray investigation of atomic vibrations in cadmium, An 73
x-ray study of lattice-distortion in copper, nickel and rhodium, An 432

Yousef, Y. L., *see* Sherbini, M. A. El

Zinc near the K absorption edge, a note on the dispersion of the atomic scattering factor of 668

CAMBRIDGE: PRINTED BY WALTER LEWIS, M.A., AT THE UNIVERSITY PRESS

157